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Non-Traded Goods in An Asymmetric Trade Model

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I. Introduction

Models with asymmetric production patterns in the countries that are participating in trade offer a framework in which the countries, unlike in the traditional trade models, do not produce exactly the same goods, but have overlapping ranges of specialization. This is a convenient way to introduce structural differences between the trade partners, and, for this reason, this notion of asymmetry has been used as the basis for "North — South" models, beginning with the work of W.A. Lewis (1954). On the other hand, in a totally different context, models with non-traded goods are becoming increasingly popular lately, especially after the contributions of Komiya (1967) and McDougall (1970). Their main analytical feature is the addition of a strictly domestic market for one or more goods to the traded goods market. Although these models do not present serious conceptual difficulties, their structure can sometimes become complicated. Jones noticed that "a general treatment of the three-commodity case [makes] it more difficult to analyze the interactions between markets." (Jones, 1974, p. 122).

Unlike in the conventional models, where the non-traded good has to be added to the group of goods produced and exchanged in the world market, non-traded goods can be readily introduced in an asymmetric structure, such as the simple setup in Lewis (1969) and (1978), or Findlay (1984). There we have a "North" producing "Food" and "Manufactures" and a "South" producing "Food" and "Coffee". In that model Lewis, letting "Food" be freely traded , proves that increases in the productivity in "Food" ("Coffee") end up unambiguously improving (worsening) the terms of trade for the "South". This

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result illustrates the mechanics of unequal exchange in a factoral terms of trade sense.

This paper combines asymmetric structure and non-traded goods in a rigorous general equilibrium framework that is easier to handle than conventional three-good models. We will build the model and show how the determination of equilibrium can be pictured in a simple diagram. Furthermore, comparative statics in this context will prove that the productivity effects on the terms of trade are, in principle, ambiguous, and, for reasonable parameter values, the Lewis results may not hold. The model is described in the first part, equilibrium determination in the second and the comparative static exercises in the third.

II. The Model

Our world economy consists of two countries or regions, the "home" and the rest of the world ("ROW"). There are three goods 1, 2 and 3, produced pairwise asymmetrically in the two countries: the home country produces goods 1 and 2 and the ROW goods 2 and 3. We will use simple Ricardian production technology for two reasons. First, because this makes the determination of equilibrium much easier. Second, because we want to maintain a structure similar to Lewis' in order to re-examine his comparative static results in a model with non-traded goods. The production functions are, therefore simple one-factor with fixed coefficients of the form:

$$Q_i = a_i L_i, \quad i = 1, 2, 3,$$

where the a's are the production coefficients (marginal and average product of labor) and the L_i's are the part of the total labor force employed in the production of good i. Full employment, assumed in both countries, gives:

$$L_1 + L_2 = L, \text{ for the home country, and}$$
$$L_3^{ROW} + L_2^{ROW} = L^{ROW} \text{ for ROW.}$$

These conditions generate the familiar linear Ricardian production possibility frontiers, with slopes equal to the ratio of the production coefficients.

On the consumption side, both countries maximize social utility functions of the form: $U(q_i)$, where q_i is the amount of good i consumed. We will assume the usual convexity properties for the perference sets, so that we can get well behaved demand functions:

$$D_i(Y, p_i) \text{ for the home country, and}$$
$$D_i^{ROW}(Y^{ROW}, p_i) \text{ for the ROW.}$$

We assume that good 2,the commonly produced one, is prevented from being traded by high transportation costs or policy measures. The trade pattern between the two countries is then clear: the home country will export good 1, and import good 3. Furthermore, since both countries consume all three goods, the case of complete specialization for either of the trade patrners is ruled out.

To determine the equilibrium, we let good 2 be the numeraire. Since, however, production and consumption technologies are not, in general, identical in the two countries, and there is no arbitrage through trade in good 2, its price need not be equalized in the home country and the ROW. We will, therefore, set the price of good 2 in the ROW equal to one, and let p_2 denote the relative price of good 2 produced at home. Notice also that, since incomplete specialization prevails, the relative prices of the country-specific goods with respect to the domestically produced good 2 are equal to the inverse of the ratio of the production coefficients. Formally:

$$Y = a_2 L, \quad Y^{\mathrm{ROW}} = a_2^{\mathrm{ROW}} L^{\mathrm{ROW}}, \quad \text{and}$$

$$p_1/p_2 = a_2/a_1, \; p_3 = a_2^{\mathrm{ROW}}/a_3^{\mathrm{ROW}}.$$

The static world trade equilibrium then is a vector of prices p_i which clears markets in all three goods and keeps the trade balanced.

III. The Static Equilibrium

a. Equilibrium in the Domestic Market

We will concentrate on the home country. If the home country is "small", the problem is obviously trivial: if the terms of trade, p_3/p_1, are given, then, since p_3 and p_1/p_2 are fixed, prices and incomes are directly determined. Hence, the levels of demands can be specified, and they, in turn, determine the allocation of labor in the production of goods. The interesting case is when the home country is treated as "large".

The market clearing conditions at home are:

$$D_2(Y, p_i) = a_2 L_2 \tag{1}$$

$$D_1(Y, p_i) + D_1^{\mathrm{ROW}}\,(Y^{\mathrm{ROW}}, p_i) = a_1 L_1 \tag{2}$$

We want, however, trade to be balanced. This implies:

$$D_1^{\text{ROW}}(\ldots) = (p_3/p_1)D_3(\ldots)$$

Substituting in (2), and using the full employment condition, we get:

$$D_1(Y, p_i) + (p_3/p_1)D_3(Y, p_i) = a_1(L - L_2) \tag{2.a}$$

Notice now that, since p_3 and p_1/p_2 are fixed and income at home depends on p_2, (1) and (2.a) form a 2×2 system in p_2 and L_2. This can most naturally be interpreted as the domestic market for the non-traded good 2. Equilibrium in this market will determine the allocation of labor at home and the price p_2, which, in turn, will tie down the rest of the relative prices in the world economy. Developing the model along these lines will give us significant intuition and permit us to illustrate the workings in a simple diagram. Some additional notation is necessary at this point: upper case Q_i's and lower case q_i's denote production and consumption levels of good i respectively; E_i's denote home income in physical terms of good i, or the endowment of good i, namely a_iL; $e_{i,j}$'s are the demand elasticities of good i with respect to the price of good j, with their original (positive or negative) signs, and n_i is the income elasticity of demand for good i; finally, the relative price ratio at home will be written as:

$$\frac{p_2}{p_1} = \frac{a_1}{a_2} = k \tag{3}$$

The domestic demand for good 2 is given in (1). To determine the slope:

$$D_2' = \frac{\partial D_2}{\partial p_1}\frac{1}{k} + \frac{\partial D_2}{\partial p_2} + \frac{\partial D_2}{\partial Y} a_2L \tag{4}$$

Substituting for k from (3), and after some standard manipulations, we get:

$$D_2' = \frac{D_2}{p_2}(e_{2,1} + e_2 + n_2) \tag{4.a}$$

Obviously the slope depends on the sign of the expression in parentheses. This peculiar demand curve can be explained if we remember that changes in the price p_2 affect demand for good 2 in three ways in this model: through the own-price effect, captured by e_2; through the cross-price effect of p_1, since p_1 and p_2 are tied together by the relative productivity ratio, captured by $e_{2,1}$; and through the income effect, since income depends on p_2, captured by h_2. This means that we may very well have a positively sloping demand curve, even if good 2 is not Giffen.

Some additional light can be shed if we use the Slutsky equation to expand (4):

$$D_2' = \frac{\partial D_2}{\partial p_1}\frac{1}{k} + \frac{\partial D_2}{\partial p_2}\bigg|_U - q_2\frac{\partial D_2}{\partial Y} + \frac{\partial D_2}{\partial Y}a_2L$$

$$= \frac{\partial D_2}{\partial p_1}\frac{1}{k} + \frac{\partial D_2}{\partial p_2}\bigg|_U + \frac{\partial D_2}{\partial Y}(E_2 - q_2) \tag{4.b}$$

In (4.b) there are two income effects on demand: the Slutsky/direct income effect, depending on the amount of good 2 consumed, and the additional indirect one described above, which depends on the endowment of good 2. These two effects work in opposite directions: the higher the consumption of 2 relative to the endowment, the closer to zero the term $(E_2 - q_2)$ is, and the less important for the slope of the demand curve the overall income effect is.

Finally, (4.a) can be drastically simplified if we recall that indirect utility and demand functions are homogeneous of degree zero in income and prices. This means that:

$$e_2 + e_{2,1} + e_{2,3} + n_2 = 0$$

Substituting, we can rewrite (4.a) as:

$$-\frac{D_2}{p_2}e_{2,3} \tag{4.c}$$

The slope here seems to depend solely and inversely on the cross-elasticity of the demand for good 2 with respect to the price of good 3. This somewhat surprising result can be readily explained: when p_2 increases (decreases), both p_1 and income increase (decrease) at the same proportion; this is equivalent to a decrease (increase) in p_3 again at the same proportion, and, thus, the total effect is equal to the cross price effect of 3 on the demand for 2.

The supply of good 2 can be derived from (2.a). Rearranging terms and multiplying both sides by a_2, we get:

$$S_2 = a_2L_2 = E_2 - \frac{a_2}{a_1}D_1(Y, p_i) - \frac{a_2}{a_1}\frac{p_3}{p_1}D_3(Y, p_i) \tag{5}$$

(5) suggests that the supply of good 2 can be thought of as a residual if, from the total endowment of 2 we subtract what is needed to satisfy demand for good 1 for consumption purposes, given the technological rate of transformation, plus demand for good 1 as a means of exchange with good 3, given the rate of transformation of one to the other through trade, namely the terms of trade. The slope of the supply curve is given by the derivative of (5), which, after some standard manipulations, can be written as:

$$S_2' = -\frac{a_2 D_1}{a_1 p_2}(e_1 + e_{1,2} + n_1) - \frac{p_3 a_2 D_3}{p_1 a_1 p_2}(e_{3,1} + e_{3,2} + n_3 - 1) \tag{6}$$

The intuition behind (6) is the following: when p_2 increases (decreases), p_1 has to increase (decrease) by a constant factor $1/k$, if equilibrium in the domestic market is to be preserved, and also home income increases (decreases); these changes affect the demands for goods 1 and 3 through the respective own and cross-price as well as income effects, and, consequently, the residual supply of good 2. Obviously, the slope of the supply curve depends on the sign of the expression (6).

Exploiting again the homogeneity property, we can get:

$$S_2' = \frac{a_2 D_1}{a_1 p_2} e_{1,3} + \frac{a_2 p_3 D_3}{a_1 p_1 p_2}(1 + e_3) \tag{6.a}$$

The demand and supply curves, given in (1) and (5) respectively, determine an equilibrium (p_2^*, q_2^*) and, since 2 is non-traded and $q_2 = Q_2 = a_2 L_2$, also L_2. It is clear from the previous analysis that the curves need not have the conventional slope. This does not make any difference for the workings of the model, as long as there exists some price p_2 at which the domestic market clears. We will, however, want to impose the assumption that the equilibrium is stable, so that our comparative static exercises are meaningful. For Walrasian stability we need the excess supply to be an increasing function of price, or, in other words:

$$S_2' > D_2' \text{ or, in this case:}$$

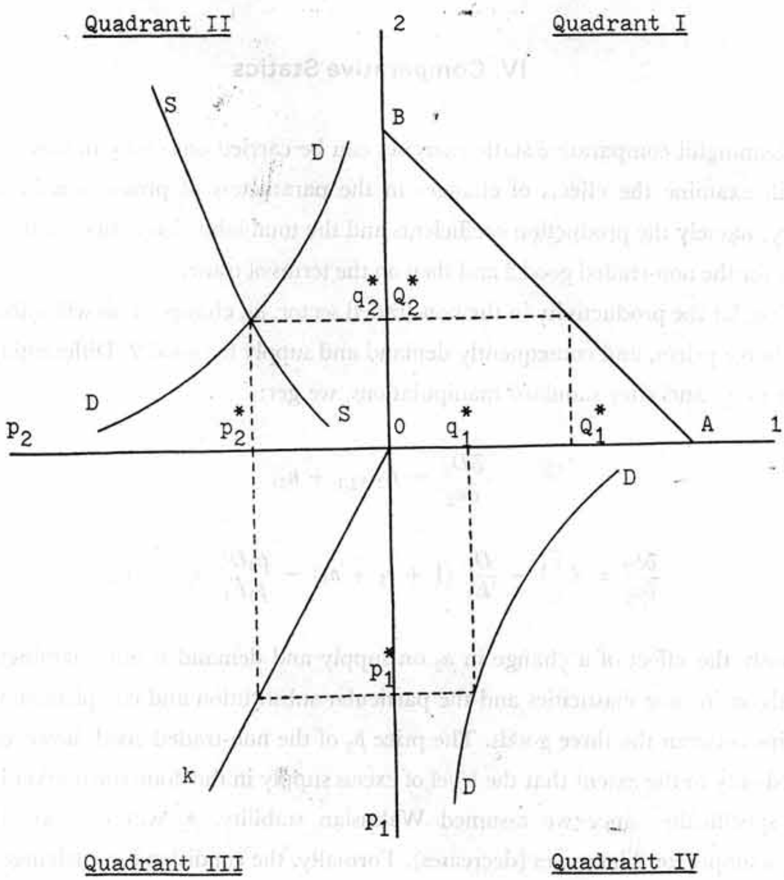
$$\frac{a_2}{a_1}\left[\frac{p_3}{p_1} D_3 (1 + e_3) + D_1 e_{1,3}\right] > D_2 e_{2,3} \tag{7}$$

Inflection points in one or both of the curves may create multiple equilibria, but we will ignore this possible complication.

b. World Trade Equilibrium

The analysis in a previous section can be summarized in a conventional demand and supply diagram. Here we will show how it is possible to picture the mechanics of global trade equilibrium in a simple geometric way. In the first quadrant of Figure 1 we have the production possibility frontier of the home country, AB. According to our notational convention, $OA = E_1$ and $OB = E_2$. In quadrant II we have drawn the supply and demand schedules for good 2 in the price-quantity space; the curves drawn here have the

Figure 1



conventional shape for convenience. In quadrant III the relationship between p_1 and p_2 is depicted; the line Ok has obviously a slope of $1/k = a_2/a_1$. Thus, the fourth quadrant is the price-quantity space for good 1, and we can draw the domestic demand for 1 as a function of its price. Again, the curve need not have the usual shape, but here it is drawn downward sloping.

Market clearing in quadrant II determines p_2^* and $q_2^* = Q_2^*$ and, consequently, Q_1^* in quadrant I. Price p_2^* is translated into p_1^* in quadrant III, and, given the demand schedule for good 1, consumption of good 1, q_1^*, is determined. The difference $Q_1^* - q_1^*$ is the amount of good 1 exported from the home country in exchange for good 3. This level of exports guarantees balanced trade by virtue of the properties of the supply curve for good 2. Finally, the allocation of labor in the ROW is determined directly from the demand function for non-traded good 2 there, since all prices in the world economy are now given.

IV. Comparative Statics

Meaningful comparative static exercises can be carried out easily in this framework. We will examine the effects of changes in the parameters of production in the home country, namely the production coefficients and the total labor force, first on the domestic market for the non-traded good 2 and then on the terms of trade.

First, let the productivity in the non-traded sector, a_2, change. This will affect income and relative prices, and consequently demand and supply for good 2. Differentiating with respect to a_2, and after standard manipulations, we get:

$$\frac{\partial D_2}{\partial a_2} = L_2(e_{2,1} + n_2) \tag{8}$$

$$\frac{\partial S_2}{\partial a_2} = L\left[1 - \frac{D_1}{E_1}(1 + e_1 + n_1) - \frac{p_3 D_3}{p_1 E_1}(e_{3,1} + n_3)\right] \tag{9}$$

Obviously the effect of a change in a_2 on supply and demand is not unambiguous, but depends on income elasticities and the particular substitution and complementarity relationships between the three goods. The price p_2 of the non-traded good, however, will be affected only to the extent that the level of excess supply in the domestic market is altered. More specifically, since we assumed Walrasian stability, p_2 will decrease (increase) if excess supply for 2 increases (decreases). Formally, the condition for unchanged p_2 is:

$$\frac{L_2}{L}(e_{2,1} + n_2) = 1 - \frac{D_1}{E_1}(1 + e_1 + n_1) - \frac{p_3 D_3}{p_1 E_1}(e_{3,1} + n_3) \tag{10}$$

To examine the effects on p_1 we can refer again to Figure 1: if a_2 increases, for example, the ray Ok in the third quadrant rotates around 0 towards the vertical axis, since now good 1 is more expensive in terms of good 2. This means that for a given p_2, p_1 is now higher. Hence, if equilibrium p_2^* in the domestic market increases or remains unchanged, p_1^* unambiguously increases and, consequently, since p_3 is fixed, the terms of trade for the home country improve. This is the typical Lewis result of an increase in productivity in "Food", valid even if "Food" is not traded. It is, however, possible, in this context, that an increase in a_2 increases excess supply of 2 and depresses p_2 so much that the new p_1^* is smaller than before, and the terms of trade worsen. We can sketch a case where this happens, if we simplify (10) by assuming that the cross price effects on demand functions are negligible. In that case, (10) can be rewritten:

$$\frac{L_2}{L} n_2 = 1 - \frac{D_1}{E_1}(1 + e_1 + n_1) - \frac{p_3 D_3}{p_1 E_1} n_3 \tag{10.a}$$

We can further simplify (10.a) using a lemma derived from what is sometimes referred to as the Engel aggregation condition.

Lemma: The budget constraint for the home country is:

$$p_1 D_1 + p_2 D_2 + p_3 D_3 = Y \tag{i}$$

Totally differentiating (i), and after standard manipulations, we get the Engel aggregation condition:

$$\frac{p_1 D_1}{Y} n_1 + \frac{p_2 D_2}{Y} n_2 + \frac{p_3 D_3}{Y} n_3 = 1 \tag{ii}$$

Observe that we can write $Y = p_i E_i$. Substitute in (ii) and rewrite:

$$\frac{D_1}{E_1} n_1 + \frac{D_2}{E_2} n_2 + \frac{p_3 D_3}{p_1 E_1} n_3 = 1, \text{ or, since 2 is not traded,}$$

$$\frac{D_1}{E_1} n_1 + \frac{p_3 D_3}{p_1 E_1} n_3 = 1 - \frac{L_2}{L} n_2 \tag{iii}$$

Using (iii), we can rewrite (10.a) as:

$$-\frac{D_1}{E_1}(1 + e_1) = 0, \text{ or } |e_1| = 1 \tag{10.b}$$

What (10.b) implies is that if the (conventionally signed) price elasticity of demand for good 1 is greater (less) than unity at home, an increase in a_2 will result in excess supply (demand) in the domestic market and a fall (increase) in p_2. If the fall in p_2 is large enough, p_1 will fall as well, and the terms of trade will have worsened. It is clear that, especially when cross effects are not zero, this is a plausible scenario for reasonable parameter values. Equation (10.a) and its simplified version (10.b), can be thought of as sufficient, but not necessary, conditions for the original Lewis result to obtain.

Let us now examine the effect of a change in the production coefficient of the traded good, a_1. This will leave income, measured in terms of good 2, unchanged, but will affect the relative prices. The derivatives of the demand and supply for good 2 with respect to a_1 are:

$$\frac{\partial D_2}{\partial a_1} = -\frac{p_1}{p_2} L_2 e_{2,1} \tag{11}$$

$$\frac{\partial S_2}{\partial a_1} = \frac{p_1}{p_2} L \left[\frac{D_1}{E_1} (1 + e_1) + \frac{p_3 D_3}{p_1 E_1} e_{3,1} \right] \tag{12}$$

Again the condition for p_2 to remain the same is that the change in a_1 does not alter excess supply, namely that:

$$-\frac{L_2}{L} e_{2,1} = \frac{D_1}{E_1} (1 + e_1) + \frac{p_3 D_3}{p_1 E_1} e_{3,1} \tag{13}$$

Back in Figure 1, if a_1 increases $0k$ rotates around 0 towards the horizontal axis this time, implying that good 1 becomes cheaper relative to 2. Hence, if p_2^* falls or remains unchanged, p_1^* and the terms of trade will be unambiguously lower than before. When, however, p_2 increases enough to offset the fall in the relative price of good 1, the terms of trade can actually improve for the home country. In that case the conclusion of the Lewis model about the negative effect on the terms of trade of an increase in the productivity in "Coffee" does not hold.

If we set again the cross effects equal to zero and try to derive the conditions which determine the behavior of excess supply in the domestic market when a_1 increases, we get, not surprisingly:

$$|e_1| = 1 \tag{13.a}$$

Equation (13.a), when satisfied, implies that p_2 remains unchanged when a_1 increases, and, by the same token, is a sufficient, but not necessary, condition for the terms of trade to worsen. When the price elasticity of the domestic demand for good 1 is so much higher than unity that the excess demand created in the domestic market offsets the effect of the falling relative price of good 1, the terms of trade can actually improve for the home country.

Finally, we will examine the effect of an increase in total labor force, L. The derivatives of the domestic demand and supply functions are:

$$\frac{\partial D_2}{\partial L} = a_2 \frac{L_2}{L} n_2 \tag{14}$$

$$\frac{\partial S_2}{\partial L} = a_2 \left(1 - \frac{D_1}{E_1} n_1 - \frac{p_3 D_3}{p_1 E_1} n_3 \right) \tag{15}$$

Using the lemma established above, we see that:

$$\frac{\partial D_2}{\partial L} = \frac{\partial S_2}{\partial L}$$

in all cases, namely an increase in the home labor force leaves the domestic excess supply and, consequently, p_2^* unchanged. Hence, since the ratio of production coefficients does not change, p_1^* and the terms of trade and also unaffected. This outcome merely confirms the initial intuition that changes in productive capacity do not have an effect on relative prices.

VI. Summary and Conclusions

We developed a simple trade model, where the partners are characterized by an asymmetric specialization pattern and, furthermore, the commonly produced good is non-traded. Concentrating on the domestic market for the non-traded good turned out to be a useful approach, since it enabled us to give an intuitive geometric exposition to the mechanics of equilibrium determination.The reader will certainly have noticed that this represents an inversion of the approach usually taken in other models with non-traded goods, where equilibrium in the traded goods determines the price in the domestic market; here, it is the domestic market which is analytically dominant and ties down the terms of trade.

Using, in particular, the structure of the Lewis model, we showed that if we allow "Food" to be non-traded, we have to part with the convenient generality of his comparative static results. It is possible that an increase in the productivity in "Food"("Coffee") in the "South" actually worsens (improves) the terms of trade with the "North". Furthermore, in the special case where cross-effects are zero or negligible, the condition for this to happen is a high demand elasticity for the traded good in the "South".

A simple intuitive argument behind this result can easily be sketched; we should keep in mind, however, that enlightening as intuition is, it can never capture the complexity of the comparative static mechanics. Let us examine the case of an increase in the technological coefficient in the production of "Food". When productivity in "Food" increases, the relative price and the quantities of the goods produced in the "South" change automatically, by virtue of the linear production possibility frontier. "Food" is now relatively cheaper and more abundant, and "Coffee" relatively more expensive. Their prices are still tied together and bound to change in the same direction, but for every level of p_2 there is now a p_1 higher than before. This is the typical Lewis' case. In our model, however, this is not the end of the story. Since "Food" is non-traded, its price is affected by the changes in supply and demand in the domestic market caused by the initial jump in productivity. The change in the relative price implies that demand for "Food" increases and demand for "Coffee" falls. The latter causes resources to be reallocated to the "Food" sector, and the supply of "Food" to increase even more than the initial increase triggered by higher productivity. If the price elasticity of "Coffee" is very high, the higher price will depress demand greatly, cause a considerable reallocation of labor to the other sector and lead to an increase in the supply of "Food" significantly larger than the initial increase in demand for "Food". The resulting excess supply can then depress p_2 enough, so that the ensuing fall in p_1 more than offsets the initial increase, and the terms of trade are worse than before. The market forces, in other words, offset the initial impact of technological change. Exactly the same line of reasoning applies, of course, in the case of an increase in productivity in "Coffee", which, under similar conditions, can lead to an improvement in the terms of trade.

Finally, it should be noted that these results are important not only for analysis. From a policy point of view, the "South" will find that the only way to improve the terms of trade is to increase productivity in "Coffee", if the domestic demand for it is very price elastic and "Food" is not traded. To put it differently, refraining from trade in "Food" may turn out to be beneficial for the terms of trade given a high elasticity of demand for the traded good and a gap in relative productivities in the commonly produced good between "North" and "South".

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