

Government Debt and the Real Exchange Rate in an Overlapping Generations Model

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Abstract

This paper examines the steady-state effect of government debt on the real exchange rate within a two-country overlapping generations model with production. It is demonstrated that, other things being constant, an increase in government debt depreciates the real exchange rate of the country with relatively higher capital elasticity of output, while it appreciates the real exchange rate of the country with relatively lower capital elasticity of output.

1. Introduction

Recently there has been growing interest in the use of microeconomic models to analyze the real exchange rate. According to Dornbusch [1989], the microeconomic framework, which emphasizes the determination of real exchange rates by resource endowments, tastes, technologies, and intertemporal choices, is appropriate to the analysis of real exchange rates in the long-run. This paper intends to examine the steady-state effect of government debt on the real exchange rate within an overlapping generations (OLG) model which is firmly grounded on microeconomic foundations.¹

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1. The OLG model was introduced by Samuelson [1958]. Diamond [1965] advanced the model by introducing capital accumulation to study the long-run effect of government debt. Recent examples for the use of OLG models include Enders and Lapan [1983], Dornbusch [1985], Persson [1985], Persson and Svensson [1985], Zee [1987, 1988], and Buiter and Kletzer [1992].

The OLG model has been widely used in many areas of economic analyses. But the use of the OLG model in the study of exchange rates is still limited. Kareken and Wallace [1981] developed an OLG model to show that in a two-country pure exchange economy the nominal exchange rate is indeterminate, in the absence of legal restrictions on portfolios. Zee [1987] developed a two-country two-good OLG model to examine the effects of government debt on capital accumulation and the terms of trade. He established that the effect of government debt on capital accumulation in each country is negative while the effect on the terms of trade is ambiguous. Ljungqvist [1988] constructed a two-country one-good OLG model to study nominal and real exchange rate volatility related to currency speculation. Since both countries produce one and the same tradable good, the real exchange rate is defined as the ratio of the real wage rates between the two countries. His model does not incorporate government debt.²

All the above studies disregarded the difference in production function among countries. To examine the steady-state effect of government debt on the real exchange rate, this paper develops a two-country OLG model in which the two countries have different production technology. Like Ljungqvist [1988], in this paper the real exchange rate is defined as the ratio of the foreign wage rate to the domestic real wage rate. The assumptions of finite life span, no bequest motive, and heterogeneous production technologies between the two countries, result in a departure from the Ricardian equivalence proposition, and permit a meaningful examination of the effect of government debt on the real exchange rate. Section II describes the model. Section III examines the effect of government debt in the steady state. Section IV is the conclusion.

II. Model

There are two countries (a domestic country and a foreign country) which are identical in every respect except for their production functions. There is only one good, which can be either consumed or invested and transported

2. Frenkel and Razin [1986a, 1986b] studied the effect of fiscal policies on the real exchange rate in a two-period equilibrium model, which does not allow the analysis of the steady-state effect of fiscal policies.

costlessly. Capital is perfectly mobile, *i.e.*, trade occurs between the two countries and imports of the good can be either invested or consumed. The labor force is constant and immobile, *i.e.*, firms in each country are allowed to use only that country's labor force. Individuals live for two periods and are identical within and across generations. In the first period they work for a firm which is owned by individuals in their second period of life. In the second period, they retire and consume their savings and accrued interest. Each of them has one unit of labor, which is supplied inelastically. There are many competitive firms in the economy and all firms within a country are identical. Firms rent capital from the old generation and employ the labor of the young.

The output of a firm depends on its employment of labor and capital (which does not depreciate). The production function of a firm in each country is specialized to the Cobb-Douglas form.³ Let y_t be output, k_t and ℓ_t be the capital and labor used in production, and A_t be the efficiency coefficient (representing the level of technology). The representative domestic firm's produced output is

$$y_t = f(k_t, \ell_t) = A_t k_t^\delta \ell_t^{1-\delta} \quad (1)$$

where $0 < \delta < 1$. ℓ_t can be normalized to one. Factor markets are perfectly competitive and each factor is paid its marginal product, thus

$$w_t = A_t (1 - \delta) k_t^\delta \quad (2)$$

$$r_t = A_t \delta k_t^{\delta-1} \quad (3)$$

where w_t is the domestic real wage rate and r_t is the real rate of return on the domestic capital.

A carat is used to indicate a corresponding foreign variable. The foreign labor force, $\hat{\ell}$ can also be normalized to one. Hence, the foreign output, \hat{y}_t , the real wage rate, \hat{w}_t , and the real rate of return on capital, \hat{r}_t are as follows:

3. Douglas [1976] argued that production processes are well described by a Cobb-Douglas function and presented evidence of the capital elasticities of output in the countries, such as Australia, Canada, New Zealand, and United Kingdom. The factor elasticities of output were apparently different among these economies.

$$\hat{y}_t = f(\hat{k}_t, \hat{\ell}_t) = \hat{A}_t \hat{k}_t^\alpha \hat{\ell}_t^{1-\alpha} \quad (4)$$

$$\hat{w}_t = \hat{A}_t (1 - \alpha) \hat{k}_t^{\alpha} \quad (5)$$

$$\hat{r}_t = \hat{A}_t \alpha \hat{k}_t^{\alpha-1} \quad (6)$$

where \hat{k}_t is the capital, and \hat{A}_t is the efficiency coefficient for the foreign country, and $0 < \alpha < 1$. In the Cobb-Douglas production functions expressed in equations (1) and (4), δ and α are the capital elasticities of output (the capital shares of output), and $(1 - \delta)$ and $(1 - \alpha)$ are the labor elasticities of output (the labor shares of output). The production functions are different if $\delta \neq \alpha$, or if $\hat{A}_t \neq \hat{A}_t$. The first inequality implies that the change in the capital stock in one country has larger impact on the output than the change in the capital stock in the other country, while the second inequality indicates that capital and labor are more efficient in one country than in the other.

By considering labor in each country as a nontraded good, we can arrive at the following definition of real exchange rates in period t , e_t , which is the ratio of the real wage rates:

$$e_t = \hat{w}_t / w_t \quad (7)$$

This definition of the real exchange rate was used by Ljungqvist [1988]. He multiplies the nominal exchange rate (the ratio of the domestic price of the tradable good to the foreign price of the tradable good) by the ratio of nominal wages in the two countries to obtain the real exchange rate. Institutions such as the International Monetary Fund have been using real wage rates as

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4. This definition of real exchange rate is consistent with those in standard textbooks. For example, in Dornbusch and Fischer [1990, pp. 184-5] the real exchange rate, e , is defined as: $e = E\hat{p}/p$, where E stands for the nominal exchange rate (price of foreign currency in terms of the domestic currency), and \hat{p} and p are the foreign and the domestic price levels. Since there are two goods in each country (the traded good and labor), the price level can be considered as a geometric average of the prices of the two goods. Let π be the price of the traded good, and $(w\pi)$ be the nominal wage rate in the domestic country. Thus, $p = \pi^\delta (w\pi)^{1-\delta}$. Assuming the same weight parameter, σ , for the foreign country gives: $\hat{p} = \hat{\pi}^\delta (\hat{w}\hat{\pi})^{1-\delta}$. In equilibrium, $E = \pi / \hat{\pi}$ (see Kareken and Wallace [1981], and Ljungqvist [1988]). Simple substitutions will yield $e = \hat{w} / w$.

measures of the real exchange rate.⁴

The domestic government finances its spending by collecting lump-sum taxes and issuing bonds. The government budget constraint is:

$$g_t + b_t(1 + r_t^b) = \tau_t + b_{t+1} \quad (8)$$

where g_t is government spending in period t ; b_t is the amount of government bonds in period t ; τ_t is lump-sum tax in period t ; r_t^b is the interest rate on the government bonds from $t-1$ to t . Without loss of generality, we assume that the foreign government spending, bonds, and taxes, are equal to zero.

Individuals pay taxes in the first period of their life, and make savings decisions at the end of this period. There are three assets for them to hold: domestic capital, foreign capital and domestic government bonds and they can hold any amount of each asset subject to their private budget constraints. The representative domestic individual maximizes the utility of consumption as follows:

$$\text{Max } U = \log c_t^t + \beta \log c_{t+1}^t$$

$$\text{s.t. } c_t^t = w_t - \tau_t - b_{t+1}^h - k_{t+1}^h - \hat{k}_{t+1}^h$$

$$c_{t+1}^t = (1 + r_{t+1}^b)b_{t+1}^h + (1 + r_{t+1})k_{t+1}^h - (1 + \hat{r}_{t+1})\hat{k}_{t+1}^h$$

where c_j^t is consumption in period j of an individual born in period t , $j = t, t+1$; b_{t+1}^h , k_{t+1}^h , and \hat{k}_{t+1}^h are the domestic government bonds, capital, and the foreign capital held by the representative domestic individual in period $t+1$, respectively. Assume that the representative foreign individual faces a similar maximization program except that taxes are zero.⁵

Solving the optimization program for both the domestic and the foreign individuals, we obtain optimal savings, s_t , and \hat{s}_t , for the domestic and the foreign individuals, respectively, and the arbitrage conditions in the asset markets:

5. Letting b_{t+1}^f , k_{t+1}^f , \hat{k}_{t+1}^f be the domestic government bonds, capital, and the foreign capital held by the representative foreign agent, yield: $k_{t+1} = k_{t+1}^h + k_{t+1}^f$, $\hat{k}_{t+1} = \hat{k}_{t+1}^h + \hat{k}_{t+1}^f$, and $b_{t+1} = b_{t+1}^h + b_{t+1}^f$.

$$s_t = (w_t - \tau_t)\beta / (1 + \beta) \quad (9)$$

$$\hat{s}_t = \hat{w}_t \beta / (1 + \beta) \quad (10)$$

$$r_{t+1} = r_{t+1}^b \quad (11)$$

$$\hat{r}_{t+1} = r_{t+1} \quad (12)$$

Equations (9) and (10) state that an individual's saving depends on his/her disposable income and time preference. Equations (11) and (12) imply that the rates of return on the domestic capital and the foreign capital must be equal to the rate of return on the government bonds. In view of equation (9), taxes are required to be less than wages at all time, *i.e.*, $\tau_t < w_t$.

A competitive equilibrium is a set of quantities $\{y_{t+1}, \hat{y}_{t+1}, k_{t+1}, \hat{k}_{t+1}, b_{t+1}, g_t, \tau_t, r_{t+1}, \hat{r}_{t+1}, r_{t+1}^b, w_{t+1}, \hat{w}_{t+1}, s_{t+1}, \hat{s}_{t+1}, e_{t+1}\}$ satisfying equations (1) – (12), and

$$\frac{[w_t - g_t - b_t(1 + r_t^b) + b_{t+1} + \hat{w}_t]\beta}{1 + \beta} = k_{t+1} + \hat{k}_{t+1} + b_{t+1} \quad (13)$$

Equation (13) states that total savings are equal to the value of total assets (the sum of the domestic and the foreign capital and the domestic bonds). Using equations (2), (3), (5), (6), and (12), equation (7) can be written as:

$$e_{t+1} = \frac{\hat{A}(1 - \alpha) \left(\frac{r_{t+1}}{\hat{A}\alpha} \right)^{\alpha/(\alpha-1)}}{A(1 - \delta) \left(\frac{r_{t+1}}{A\delta} \right)^{\delta/(\delta-1)}} = \frac{\hat{A}(1 - \alpha) (A\delta)^{\delta/(\delta-1)}}{A(1 - \delta) (\hat{A}\alpha)^{\alpha/(\alpha-1)}} r_{t+1}^{\frac{\alpha}{\alpha-1} - \frac{\delta}{\delta-1}} \quad (14)$$

In the steady state, $k_t = k$, $\hat{k}_t = \hat{k}$, $r_t = r$, $w_t = w$, $\hat{w}_t = \hat{w}$, $e_t = e$, $b_t = b$, $g_t = g$ and $\tau_t = \tau$. The steady-state versions of equations (2), (3), (5), (6), (8), (13) and (14) are as follows:

$$A(1 - \delta)k^\delta = w \quad (15)$$

$$A\delta k^{\delta-1} = r \quad (16)$$

$$\hat{A}(1 - \alpha) \hat{k}^\alpha = \hat{w} \quad (17)$$

$$\hat{A} \alpha \hat{k}^{\alpha-1} = r \quad (18)$$

$$g + br = \tau \quad (19)$$

$$\frac{[A(1-\delta)\hat{k}^\delta - g - br + \hat{A}(1-\alpha)\hat{k}^\alpha]\beta}{1+\beta} = \hat{k} + \hat{k} + b \quad (20)$$

$$e = \frac{\hat{A}^{1/(1-\alpha)} (1-\alpha)}{A^{1/(1-\delta)} (1-\delta)} \frac{\delta^{\delta/(\delta-1)}}{\alpha^{\alpha/(\alpha-1)}} r^{\frac{\alpha}{\alpha-1} - \frac{\delta}{\delta-1}} \quad (21)$$

In this model, it is not possible for the government to run Ponzi game schemes (in which all principal repayments and interest on debt are financed by issuing new debt) because the real interest rate is greater than the growth rate of the labor force (equal to 0 here). From equation (19), government debt, b , can be either positive or negative. If the government debt is positive, then the government must run a steady-state budget surplus (*i.e.*, $g < \tau$) to pay all the interest on its debt; if the steady-state government debt is negative, then steady-state budget deficits must be greater than 0 (*i.e.*, $g > \tau$), and the government is lending to earn interest to finance the deficits. Therefore, in the steady-state, government cannot borrow to finance its budget deficits and interest on debt. This model is similar to the models considered by Persson [1985] and Zee [1987], where they assumed that the growth rate of the labor force is positive but less than the real interest rate. In Diamond's [1965] OLG model, if the steady-state interest rate is less than the growth rate of the labor force (dynamically inefficient), then the steady-state government debt and deficits can both be positive, and thus, Ponzi games are indeed possible (see O'Connell and Zeldes [1988] for a discussion of the general conditions that make Ponzi games feasible), *i.e.*, new debt can finance all of the interest payments on the existing debt plus some additional transfers to the young.

III. The Comparative Steady-State Analyses

This section examines the steady-state effects of budget deficits on the real interest rate and the real exchange rate. Assume that government spending is constant and the government sets taxes so as to keep debt per worker

constant in the long run.⁶ Condensing equations (15)-(21) into one equation yields:

$$\frac{\left[A(1-\delta)k^\delta - g - bA\delta k^{\delta-1} + \hat{A}(1-\alpha) \left(\frac{A\delta}{\hat{A}\alpha} \right)^{\alpha/(\alpha-1)} k^{\frac{\alpha(\delta-1)}{\alpha-1}} \right] \beta}{1+\beta} - k - \left(\frac{A\delta}{\hat{A}\alpha} \right)^{\frac{1}{\alpha-1}} k^{\frac{\delta-1}{\alpha-1}} - b = 0 \quad (22)$$

Totally differentiating (22) with respect to k and b gives:

$$\frac{dk}{db} = \frac{\frac{\beta}{(1+\beta)} A\delta k^{\delta-1} + 1}{\Omega}$$

where

$$\Omega = \frac{\left[\beta A\delta(1-\delta)k^{\delta-1} - bA\delta(\delta-1)k^{\delta-2} + \hat{A}(1-\alpha) \left(\frac{A\delta}{\hat{A}\alpha} \right)^{\alpha/(\alpha-1)} \frac{\alpha(\delta-1)}{\alpha-1} k^{\frac{\alpha(\delta-1)-\alpha+1}{\alpha-1}} \right]}{1+\beta} - 1 - \left(\frac{A\delta}{\hat{A}\alpha} \right)^{\frac{1}{\alpha-1}} \frac{(\delta-1)}{(\alpha-1)} k^{\frac{\delta-\alpha}{\alpha-1}}$$

The numerator is always positive, and the stability condition implies that $\Omega < 0$.⁷ Thus, $dk/db < 0$. That is, an increase in government debt will lower the capital-labor ratio.

To examine the change in the real interest rate and the real exchange rate with a change in government debt, differentiate equations (16) and (21):

$$\frac{dr}{db} = A\delta(\delta-1)k^{\delta-2} \frac{dk}{db}$$

$$\frac{de}{db} = \frac{\hat{A}^{1/(1-\alpha)}(1-\alpha)}{A^{1/(1-\delta)}(1-\delta)} \frac{\delta^{\delta/(\delta-1)}}{\alpha^{\alpha/(\alpha-1)}} \left(\frac{\alpha}{\alpha-1} - \frac{\delta}{\delta-1} \right) r^{\frac{\alpha}{\alpha-1} - \frac{\delta}{\delta-1} - 1} \frac{dr}{db}$$

Since $\frac{dk}{db} < 0$, $\frac{dr}{db} > 0$, i.e., an increase in the government debt increases the real interest rate. However, the sign of $\frac{de}{db}$ depends on the sign of $\frac{\alpha}{(\alpha-1)}$

6. This assumption was also made by Diamond [1965], Blanchard [1985], Persson [1985], and Zee [1987].

7. The derivation of the stability condition is similar to that of Zee [1987] and is available from the author on request.

$-\frac{\delta}{\delta-1}$ and $\frac{dr}{db}$. It can be shown that $\frac{\alpha}{(\alpha-1)} - \frac{\delta}{\delta-1} \geq 0$ if $\delta \geq \alpha$. Therefore, $\frac{de}{db} \geq 0$ if $\delta \geq \alpha$, i.e., other things being constant, an increase in government debt depreciates the real exchange rate of the country with relatively higher capital elasticity of output, appreciates the real exchange rate of the country with relatively lower capital elasticity of output, and has no effect on the real exchange rate if the two countries have the same capital elasticity in production. The intuition is as follows. In the steady state an increase in government debt causes an increase in taxes in order to pay the additional interest. This rise in taxes lowers steady-state disposable income and thereby lowers the domestic savings. In order to restore equilibrium, capital stocks in both countries must decrease and the real interest rate must increase. However, the magnitudes of the decrease in the capital stocks in the two countries are different. This difference causes a disproportional change in the real wage rate, and therefore, a change in the real exchange rate. Note that the efficiency coefficients, A and \hat{A} are important in determining the magnitude, but not the sign, of $\frac{de}{db}$.⁸

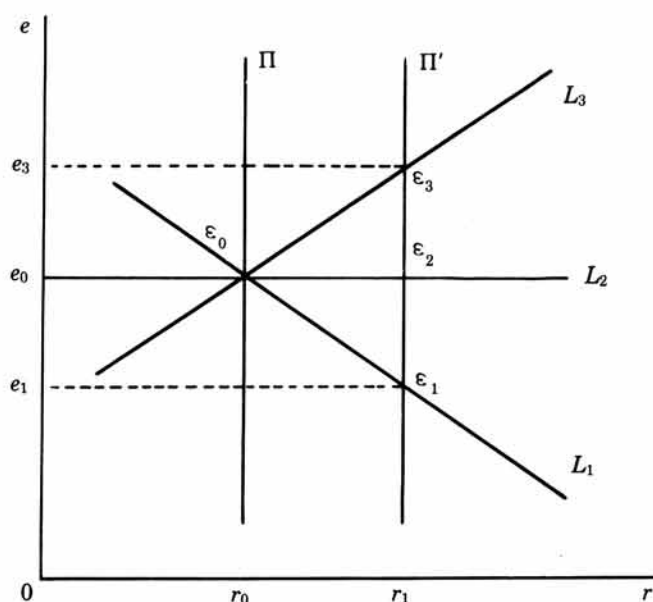
Figure 1 illustrates the effects of an increase in government debt on the real interest rate and the real exchange rate. Curves L1, L2, and L3 represent equation (21) with $\delta < \alpha$, $\delta = \alpha$, and $\delta > \alpha$, respectively. Curve Π represents equation (20) with $k = \left(\frac{r}{A\delta}\right)^{1/(\delta-1)}$ and $\hat{k} = \left(\frac{r}{\hat{A}\alpha}\right)^{1/(\alpha-1)}$. Point e_0 represents the initial equilibrium. When government debt increases, curve Π shift to Π' while curves L1, L2, and L3 remain unchanged. If $\delta < \alpha$, the real exchange rate of the domestic country will decrease from e_0 to e_1 (a real appreciation). If $\delta =$

8. An extension of the analysis is to consider a more general production technology, such as:

$$y_t = f(A_t k_t, B_t \ell_t), \quad \hat{y}_t = f(\hat{A}_t \hat{k}_t, \hat{B}_t \hat{\ell}_t), \quad f_1 > 0, f_2 > 0, f_{12} > 0, f_{11} < 0, f_{22} < 0.$$

Although it is clear from the extended analysis that the labor- and capital-augmenting coefficients, A, \hat{A}, B , and \hat{B} , are important for determining the response of exchange rates to government debt, this production function is too general to reveal the direction of response from simple differences in A and \hat{A} , and B and \hat{B} (the details are available from the author on request).

Figure 1
Effect of An Increase in the Government Debt
on the Real Interest Rate and the Real Exchange Rate



α , the real exchange rate will remain at e_0 . If $\delta > \alpha$, the real exchange rate will increase from e_0 to e_3 (a real depreciation). In all the cases, the real interest rate will increase from r_0 to r_1 .

IV. Concluding Remarks

In the context of the OLG developed here, the importance of production technologies in the determination of the effect of government debt on the long-run real exchange rate is clearly demonstrated. It should be mentioned that the present study only focuses on how government debt affects the real exchange rate through the channel of production. As Dornbusch [1989] pointed out, the long-run real exchange rates can be determined by other differences between countries as well, such as time preference and resource endowments. Thus, a natural extension would be to incorporate other differences between countries. Also, in this model production functions are differ-

ent only in the factor elasticity of output and the efficiency coefficient. Other ways to model the differences in production are possible. Finally, it would be desirable to incorporate in one model the differences between countries in more than one factor. However, the complexity of such an extension should not be underestimated.

References

- Blanchard, O. J. [1985], "Debts, Deficits and Finite Horizons," *Journal of Political Economy* 92; pp. 223-47.
- Buiter, W. H. and K. M. Kletzer (1992), "Government Solvency, Ponzi Finance and Redundancy and Usefulness of Public Debt," Economic Growth Center Discussion Paper No. 659, Yale University.
- Diamond, P. A. [1965], "National Debt in a Neoclassical Growth Model," *American Economic Review* 55; pp. 1125-50.
- Dornbusch, R. [1989], "Real Exchange Rates and Macroeconomics: A Selective Survey," *Scandinavian Journal of Economics* 91; pp. 401-32.
- Dornbusch, R. and S. Fischer [1990], *Macroeconomics*, Fifth edition, McGraw-Hill, New York.
- Douglas, P. H. [1976], "The Cobb-Douglas Production Function Once Again: Its History, Its Testing, and Some New Empirical Values," *Journal of Political Economy* 84; pp. 903-15.
- Enders, W. and H. E. Lapan [1983], "On the Relationship between the Exchange Regimes and the Portfolio Rules of Optimizing Agents," *Journal of International Economics* 15; pp. 199-224.
- Frenkel, J. A. and A. Razin [1986a], "The International Transmission and Effects of Fiscal Policies," *American Economic Review* 76; pp. 330-35.
- Frenkel, J. A. and A. Razin [1986b], "Real Exchange Rates, Interest Rates and Fiscal Policies," *Economic Study Quarterly* 37; pp. 99-113.
- Kareken, J. and N. Wallace [1981], "On the Indeterminacy of Equilibrium Exchange Rates," *Quarterly Journal of Economics* 96; pp. 207-22.
- Ljungqvist, L. [1988], "A Theoretical Contribution to Old Controversials: Does Speculation Affect Exchange Rate Volatility and Do Dual Economies Exist?" Ph.D. Dissertation, University of Minnesota.
- O'Connell, S. A. and S. P. Zeldes [1988], "Rational Ponzi Games," *International*

Economic Review 29; pp. 431-50.

Persson, T. [1985], "Deficits and Intergenerational Welfare in Open Economies," *Journal of International Economics* 19; pp. 67-84.

Samuelson, P. A. [1958], "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy* 66; pp. 467-82.

Zee, H. H [1987], "Government Debt, Capital Accumulation, and The Terms of Trade in a Model of Interdependent Economy," *Economy Inquiry* 25; pp. 599-618.

Zee, H. H [1988], "The Sustainability and Optimality of Government Debt," *International Monetary Fund Staff Papers* 35; pp. 658-85.