

## Optimal Monetary and Exchange-Rate Policy with Wage Indexation

Arthur Benavie and Richard Froyen\*

*University of North Carolina*

### Abstract

*This paper investigates the setting and coordination of monetary policy and foreign exchange market intervention. This is done within a framework that invokes neither purchasing power parity nor uncovered interest parity and which allows for wage indexation. Optimal policies are designed in the presence of domestic IS, LM and productivity shocks, as well as international asset demand shocks and foreign income, price and interest rate shocks. One feature of these policies is that for the monetary policy parameter a Poole (1970)-type ranking emerges, with a fixed interest rate optimal for all financial shocks and a vertical LM curve optimal for domestic IS and foreign income shocks. Also of interest is the fact that for both these sets of shocks a fixed exchange rate is part of the optimal policy. Only for foreign price shocks is an exchange rate adjustment part of the optimal response.*

### I. Introduction

This paper investigates the optimal setting and coordination of monetary policy and exchange rate intervention rules in an economy where wages are indexed to a general price level. We do this within a framework that does not invoke either purchasing power parity (PPP) or uncovered interest rate parity (UIP). Our analysis builds on and extends several strands of optimal stabilization policy literature as well as the literature on wage indexation. Optimal combination monetary policies, analyzed by Poole [1970], Leroy and Waud [1977] and, within a rational expecta-

---

\* Department of Economics, University of North Carolina, Chapel Hill N.C. 27599, U.S.A.

tions context, by Woglom [1979], Canzoneri, Henderson and Rogoff [1983] and Benavie and Froyen [1983] can be viewed as monetary stabilizers. Friedman [1975], Turnovsky [1980] and Craine and Havenner [1981], as well as the above studies, also consider *pure* monetary policies which peg either the interest rate or the level of a monetary or reserve aggregate. These studies are all for a closed economy where monetary policy is assumed to affect the price level and interest rate independently of the constraints of PPP and UIP. The analysis here extends this strand in the literature by adding a foreign sector.

The literature on exchange rate intervention (Boyer [1978], Roper and Turnovsky [1980] and Cox [1980]) assumes that the central bank executes contracts for foreign exchange reserves conditional on the current exchange rate. These papers all assume UIP, thus monetary stabilizers are indistinguishable from exchange rate intervention, i.e., the intervention rule determines the money supply.<sup>1</sup> Here monetary stabilizers and exchange rate intervention are considered separately in a model with less than perfect capital mobility. We also relax the assumption of either a fixed price or PPP in these models.

Other articles consider optimal monetary policy along with wage indexation. These studies include: Aizenman and Frenkel [1985, 1986], Marston [1985], Turnovsky [1983, 1984, 1987] and Devereux [1988]. Some of these papers include the exchange rate in the monetary policy rule, and therefore, consider optimal exchange rate intervention as well. All assume UIP and/or PPP.<sup>2</sup> The approach here builds on these studies but differs in that neither PPP nor UIP are invoked.<sup>3</sup>

Henderson [1979, 1982] compares an *aggregates* (money supply and private holdings of domestic securities) *constant* versus a *rates* (interest rate and exchange

- 
1. Artis and Currie [1981] consider the choice of a monetary policy that targets the exchange rate relative to one that targets a monetary aggregate. They assume UIP holds except in the presence of an interest-equalization tax. Parkin [1978] and Turnovsky [1976] consider fixed versus flexible exchange rates in an economy with imperfect capital mobility.
  2. All except Turnovsky [1983] and Devereux [1988] assume both UIP and PPP. Optimal wage indexation rules are considered apart from optimal monetary policy by Marston [1982, 1984], Marston and Turnovsky [1985], and Flood and Marion [1982].
  3. There is a parallel literature for closed economy models. Examples are, Fethke and Jackman [1984], Bradley and Jansen [1988], and Van Hoose and Waller [1989]. Other studies consider the role of wage indexation in models which include strategic considerations of the Barro and Gordon [1983] type. Examples are Devereux [1989] and Van Hoose and Waller [1991, 1992].

rate) *constant* policy as an employment stabilizer in a model without PPP or UIP and with wage indexation. Our analysis builds on his by considering a broader range of policy rules and objectives. Benavie [1983] considers optimal monetary policy and exchange rate intervention, also assuming imperfect capital mobility and without PPP. Benavie does not consider wage indexation (or labor contracts).<sup>4</sup>

Our policy set up is as follows. Monetary policy is specified in the manner of Poole's [1970] combination policy where the money supply responds to movements of the interest rate away from its target value. Analogously, central bank purchases of foreign reserve assets are assumed to respond to movements in the exchange rate relative to its target value. The degree to which these purchases of foreign reserve assets affect the money supply depends on a sterilization parameter, which is an additional policy parameter. The model used for the analysis is developed in Section II. The solution of the model is set out in Section III. Section IV discussed the policymaker's goals and information set. In Section V optimal policies are derived and discussed. Section VI summarizes our results. Section VII relates our findings to those in the literature. Section VIII contains concluding comments.

## II. The Model

The model is as follows:

$$y_t = -a_1[r_t - E_{t-1}(p_{t+1} - p_t)] + a_2(p_t^f + x_t - p_t) + a_3y_t^f + v_{1t} \quad (1)$$

$$y_t = c_0 + c_1(p_t - E_{t-1}p_t) + c_2(p_t - E_{t-1}\bar{p}_t) - c_3b(\bar{p}_t - E_{t-1}\bar{p}_t) + c_4v_{2t} \quad (2)$$

$$m_t = \gamma_1y_t - \gamma_2r_t - \gamma_3(r_t^f + E_t x_{t+1} - x_t) + p_t + v_{3t} \quad (3)$$

$$m_t \equiv \bar{m} + \Delta m \equiv \bar{m} + \theta \Delta f_t + (1 - \theta) \Delta d_t \quad (4)$$

$$\Delta f_t = -\mu(x_t - \bar{x}) \quad (5)$$

$$\Delta d_t = \lambda(r_t - \bar{r}) - \gamma \Delta f_t \quad (6)$$

$$\Delta f_t = \phi_1(p_t^f + x_t - p_t) - \phi_2y_t + \phi_3r_t - \phi_4(r_t^f + E_t x_{t+1} - x_t) + \phi_5y_t^f + v_{4t} \quad (7)$$

4. Benavie [1983] considers only nonsterilized exchange rate intervention, while here the degree of sterilization is a policy parameter.

where all variables are in natural logarithms except for interest rates and where

$y$  = real domestic output

$p$  = domestic price level

$r$  = domestic interest rate

$r^f$  = foreign interest rate

$p^f$  = foreign price level

$y^f$  = real foreign output

$x$  = exchange rate (units of domestic currency per unit foreign currency)

$\bar{p}$  = overall price index  $\equiv ap + (1-a)(p^f + x)$ ,  $0 < a < 1$

$b$  = wage indexation parameter,  $0 \leq b \leq 1$

$m$  = nominal money stock

$\Delta f$  = deviation of stock of foreign reserves from target level ( $f - \bar{f}$ )

$\Delta d$  = deviation of domestic credit from target level ( $d - \bar{d}$ )

$\theta$  = ratio of foreign exchange reserves to money

$\gamma$  = sterilization coefficient

$\bar{x}$  = long-run equilibrium value of  $x$

$\bar{r}$  = long-run equilibrium value of  $r$

$\bar{m}$  = target money stock at  $\bar{x}$  and  $\bar{r}$

$E_{t-1}$  = the conditional expectation of the indicated variable taken at  $t-1$

$v_{it}$  = white noise disturbance terms ( $i = 1, 2, 3, 4$ ).

All parameters with the exception of  $\lambda$ , and  $\mu$  have positive values. The ranges

of values for  $\lambda$  and  $\mu$  are discussed below.

Equation (1) is a standard open economy IS function. Equation (2) is the aggregate supply equation, the derivation of which follows Marston [1985] with details given in Appendix A. Here it is sufficient to note that a Cobb-Douglas production function is used to derive a labor demand schedule. Labor supply is assumed to depend on the real wage, defined as the money wage deflated by the overall price index ( $w - \bar{p}$ ). A labor contracting procedure is assumed, such that a contract wage ( $w_t^c$ ) is set for period  $t$  to equate the expected values at  $t - 1$  of labor supply and demand. The actual wage,  $w_t$ , is taken to be indexed to the overall price index

$$w_t = w_t^c + b(\bar{p}_t - E_{t-1}\bar{p}_t) \quad (8)$$

The indexation parameter is assumed to vary between zero and one.

Asset markets including the market for foreign exchange are set up in an end-of-period framework. Equation (3) is a standard money demand function, where domestic residents have two assets competitive with money – domestic and foreign bonds.<sup>5,6</sup>

Equation (4) determines the money supply, which is defined as an end-of-period stock, where  $\bar{m}$  is the target money supply for the case where  $x$  and  $r$  take on their long-run equilibrium values  $\bar{x}$  and  $\bar{r}$ . The deviations of the actual money supply from this target level,  $\Delta f_t$  and  $\Delta d_t$ , come as the result of foreign exchange market

5. In addition to the arguments in (3), the domestic demand for money, as well as the demands for both domestic and foreign bonds, depends upon wealth. Even in our end-of-period discrete time formulation of asset markets, however, the relevant asset stocks that define wealth are those at period  $(t - 1)$  (see Turnovsky [1976]). Since we consider only one period these are fixed and can be subsumed in the constant term (which is dropped from (3) since it is not used in our analysis). Wealth effects are crucial to the longer-run adjustment to stock equilibrium in asset markets, but not to our short-run analysis.
6. As a simplification, nominal money demand in (3) is specified as a function of only the domestic price level ( $p$ ), rather than the general price level ( $\bar{p}$ ). Were we to use  $\bar{p}$ , the exchange rate would enter (3) with an additional coefficient and the foreign price level would enter. The additional coefficient on the exchange rate would change the particular expressions for some of our optimal policies but would not substantively affect our results. Because the foreign price level will be specified as exogenous, its presence in (3) would have no effect except when we consider shocks to it. We return to this point when foreign price shocks are considered below.

intervention and interest rate smoothing.<sup>7</sup> Equation (5) describes exchange rate intervention policy, where the degree of intervention is a function of the deviation of the current exchange rate from its long-run equilibrium value which the authorities are assumed to know.<sup>8</sup> The limiting cases  $\mu \rightarrow \infty$ , and  $\mu = 0$  correspond to fixed and flexible exchange rate regimes, respectively, while any finite value of  $\mu$  describes a managed float.

Equation (6) specifies the interest rate smoothing rule and sterilization operations, where  $\gamma$  is the sterilization coefficient. Assuming, with no effect on our analysis, that  $\theta = 1/2$ , it follows that if  $\gamma = 1$ , there is complete sterilization, while if  $\gamma = 0$  there is no sterilization. The flow supply of money is linked (via  $\Delta d$ ) to the deviation of the current interest rate from its long-run equilibrium value. If  $\lambda = 0$  there is no interest rate smoothing, while if  $\lambda \rightarrow \infty$  we have the special case of an interest rate target.

Equation (7) describes the standard influences on the overall balance of payments arising from trade and capital account transactions.<sup>9</sup> The net trade balance is assumed to depend positively on the international relative price and foreign output and negatively on domestic output. The capital account balance is assumed to depend on both the domestic and foreign interest rate, as well as the variables that affect the net trade balance.<sup>10</sup>

We maintain the simplifying assumption that the foreign variables,  $y^f$ ,  $p^f$  and  $r^f$  are exogenous. It would be better to spell out the foreign sector. We have, however,

- 
7. Because  $f$  and  $d$  are in natural logarithms,  $m_t - \bar{m}$  is (approximately) the percentage deviation of the actual money supply from the target level.
  8. The assumption that policymakers know the long-run equilibrium exchange rate is not a trivial one. Through this assumption, we ignore the problem that if the long-run equilibrium rate is not known the monetary authority might, for example, intervene to defend a disequilibrium rate. This problem should be kept in mind in evaluating the optimal policy settings below, but our framework can say nothing about the difficulties of ascertaining the true equilibrium exchange rate. For an analysis of this question, see Krugman [1990].
  9. Benavie [1983] shows how (7) can be derived from the rest of a model of this type. Here, as in Benavie [1983], it is assumed that the current account is initially in balance and wealth effects due to changes in domestic or foreign price levels are ignored. Were these assumptions not made, the domestic and foreign price levels would appear in (7) with separate coefficients with no substantive effect on our analysis.
  10. Allowing for the effects of domestic output and domestic price on the capital account suggests possible ambiguities in the effects of these variables on the balance of payments. An increase in  $y$  for example will increase the domestic demand for money partly at the expense

in a previous version done so in a simple fashion (which followed Marston [1985] and Flood and Marion [1982]) and we find *no* policy settings that achieve the stabilization goals discussed below in the face of foreign shocks, for each foreign shock (e.g. a money demand shock) will affect all three foreign variables,  $y^f$ ,  $r^f$  and  $p^f$ . It will therefore provide more insight to assume exogeneity for these foreign variables, since if they are taken one at a time then optimal (in the sense of satisfying all our stabilization goals) policy settings do exist. Studying the latter will clarify why there is no optimal policy setting in the presence of simultaneous changes in all three foreign variables.

### III. Model Solution

Throughout, rational expectations are assumed but given the information structure we could assume exogenous or adaptive expectations with no substantive alteration in these short-run results. The first step in solving the model is to use (5) in (7), (5) and (6) in (4), and the result in (3) and solve jointly  $r$  and  $x$  (eliminating time subscripts,  $\bar{m}$ ,  $\bar{x}$ , and  $\bar{r}$ ) which gives

$$r = \alpha_{11}y + \alpha_{12}p + \alpha_{13}y^f + \alpha_{14}p^f + \alpha_{15}E_t x_{t+1} + \alpha_{16}r^f + \alpha_{17}v_3 + \alpha_{18}v_4 \quad (9)$$

$$x = \alpha_{21}y + \alpha_{22}p + \alpha_{23}y^f + \alpha_{24}p^f + \alpha_{25}E_t x_{t+1} + \alpha_{26}r^f + \alpha_{27}v_3 + \alpha_{28}v_4 \quad (10)$$

where the  $\alpha_{ij}$ 's are defined in Appendix B.

Next, insert (9) and (10) and  $\bar{p} \equiv ap + (1-a)(p^f + x)$  into (1) and (2). Let the trial solutions for  $y$  and  $p$  be

$$y = \psi_{10} + \psi_{11}y^f + \psi_{12}p^f + \psi_{13}r^f + \psi_{14}v_1 + \psi_{15}v_2 + \psi_{16}v_3 + \psi_{17}v_4 \quad (11)$$

$$p = \psi_{20} + \psi_{21}y^f + \psi_{22}p^f + \psi_{23}r^f + \psi_{24}v_1 + \psi_{25}v_2 + \psi_{26}v_3 + \psi_{27}v_4 \quad (12)$$

where the  $\psi_{ij}$ 's are yet to be determined. Using a trial solution for  $x_t$  of the same form

---

of foreign bonds. This will reduce the excess demand for foreign exchange. On the other hand, saving will rise which will increase the demand for foreign bonds, as well as other assets. These effects will be in addition to the current account effect where an increase in  $y$  leads to an increased demand for foreign exchange to finance a higher volume of imports. Our assumption will be that *ceteris paribus* of a rise in domestic output or price (given the foreign price) will increase the excess demand for foreign exchange.

as (11) or (12), it is evident that  $E_t x_{t+1}$  is a constant, as are all the expectations conditioned on lagged information. Using (11) and (12) in (9) and (10), and the latter along with (11) and (12) in (1) and (2) yields a set of identities involving the  $\psi_{ij}$ 's and the structural coefficients of the model. From these identities we obtain the values for the  $\psi_{ij}$ 's, which are written out in Appendix B (where constants, which will not be used, are ignored).

The discrepancy between output and full-information output,  $y^*$ , will also be a concern. To solve for  $y^*$ , the demand for labor is set equal to the supply to yield the equilibrium wage. The equilibrium wage is then inserted into the labor demand function and the result substituted into the production function (see Appendix A) to yield

$$y^* = \frac{1-c}{c}(1-\beta_1)p - \frac{1-c}{c}\beta_2(p^f + x) + \frac{1}{c}(1-(1-c)\beta_3)v_2 \quad (13)$$

where the  $\beta$ 's are parameters from the wage equation as defined in Appendix B and where the constant term is ignored.

## IV. The Policymaker's Environment

### A. The Policymaker's Goals

The policymaker is assumed to desire to eliminate the variance of a set of goal variables around optimal levels. Deviations of these variables from optimal levels are assumed to come about as a result of temporary shocks.<sup>11</sup>

The set of policy objectives considered here subsumes all of those which have been commonly used in the optimal policy literature. We assume that the policymaker wishes to eliminate the variance in the following variables:

1. output,  $y$
2. the domestic price,  $p$
3. the overall price index,  $\bar{p} = ap + (1-a)(p^f + x)$
4. the terms of trade,  $\varepsilon = p^f + x - p$

---

11. Given the model specification, where with the exception of  $E_t x_{t+1}$  all expectations are dated  $t-1$ , consideration of permanent shocks would differ from that of temporary shocks (for purposes of the short run) in only one respect; if shocks were permanent then current period observations on  $x$  and  $r$  would contain information about shocks which would modify  $E_t x_{t+1}$ .

5. output less full-information output,  $y - y^*$ 

Papers such as Turnovsky [1983] assume that the policymaker minimizes a loss function of the form.

$$L_1 = \sigma_1(p - \tilde{p})^2 + \sigma_2(y - \tilde{y})^2$$

The policymaker attempts to minimize the variance of price and output around the desired levels ( $\tilde{p}$ ,  $\tilde{y}$ ). Another plausible goal to add to these is to minimize the variability in the terms of trade ( $\varepsilon = p^f + x - p$ ); the gains from this are reduced adjustment costs due to terms-of-trade-induced changes in resource allocation. In addition, the domestic price level,  $p$ , is not only price index of concern to domestic agents, the overall price index,  $\bar{p} = ap + (1-a)(p^f + x)$ , is also relevant. Hence, we include the stabilization of  $i$  as an additional goal.

Alternatively, studies such as Aizenman and Frenkel [1985, 1986] focus on the goal of minimizing the variance of output around full-information output ( $y^*$ )

$$L_2 = \sigma_3(y - y^*)^2$$

This formulation has a clear basis in utility maximization because it can be shown that minimizing  $L_2$  will minimize the welfare loss from nonoptimal levels of output and employment (see Aizenman and Frenkel [1985]).

Rather than choose between the approaches that lead to  $L_1$  and  $L_2$  we elect to consider the goal variables suggested by both. We do so even though the goals in the  $L_1$  approach do not have a clear basis in utility maximization. They are, however, among the announced goals of central banks. Also, they seem to be plausible; for example, an increase in price uncertainty appears to involve a welfare loss due to planning difficulties even if output is kept equal to full-information output. We thus consider policies which minimize the variance in the above five target variables; and we define an *optimal* policy as one which completely eliminates the variance in all five of these goal variables.

### ***B. The Policymaker's Information Set***

We assume that policymakers observe (in addition to lagged variables) only the current domestic interest rate and the exchange rate. They do not observe the

current domestic or foreign price or output level. They may observe the foreign interest rate but with the exogenous specification of the foreign sector in Section II, (and in the absence of UIP) there is no gain in tying policy to the foreign interest rate.<sup>12</sup>

Labor suppliers and firms at the time the contract wage is set, condition their expectation of the current and future price level on *no* current variables. This ability of policymakers to condition policy actions on information private economic decisions were unable to employ provides the mechanism by which policy actions affect real variables. Private agents in financial markets are assumed to condition their expectations of the future exchange rate on the current domestic interest rate and the exchange rate.

## V. Optimal Policy

We now investigate optimal monetary and exchange rate intervention policies in the presence of random disturbances, given that the wage indexation parameter,  $b$ , is between zero and one, and where the sterilization coefficient,  $\gamma$ , is also taken to be between zero and one and is considered a policy parameter. Recall that  $\lambda$  is the monetary policy parameter and  $\mu$  is the exchange-rate intervention coefficient. For expositional purposes, at first shocks are considered one at a time. Analyzing one shock at a time will lead to propositions concerning optimal policy settings in the face of multiple random disturbances.

### A. IS ( $v_t$ ) or Foreign Output ( $y^f$ ) Shock

#### 1. IS ( $v_t$ ) Disturbance

For IS equation disturbances, the unique optimal policy setting is,

$$\lambda = -\gamma_2 + \phi_3(\gamma - 1) \quad \mu \rightarrow \infty \quad (14)$$

---

12. The policymaker could set up a money supply or exchange market intervention rule that was conditioned on the foreign interest rate. Such a rule could offset the effect of that shock. In our framework the same result is achieved by a feedback rule conditioned on the domestic interest rate and exchange rate.

where  $0 \leq \gamma \leq 1$ . This policy setting is optimal for  $0 \leq b \leq 1$ .

The policy setting in (14) sets  $\psi_{14} = \psi_{24} = 0$  and targets  $x$  which implies that price, output and the terms of trade are all stabilized. In addition,  $(y - y^*)$  and the overall price index are stabilized. This policy works by fixing the exchange rate and by making the LM curve vertical in both  $(r, y)$  and  $(r, p)$  space. The effect of this is to prevent aggregate supply shifts in  $(p, y)$  space by keeping  $x$  unchanged; and – as in the textbook Keynesian models – to see to it that displacements in the IS schedule imply only interest rate changes and no displacement in the aggregate demand schedule in  $(p, y)$  space. Note that the setting in (14) sets  $\alpha_{11} \rightarrow \infty$  and  $\alpha_{12} \rightarrow \infty$  i.e., the interest rate is infinitely sensitive to  $p$  and  $y$ .

## 2. A Foreign Output ( $y^f$ ) Shock

In the case of a  $y^f$  shock, (14) is also an optimal policy with complete sterilization ( $\gamma = 1$ ). If sterilization is not complete, a fixed exchange rate remains optimal but the  $\lambda$  value in (14) is replaced by

$$\lambda = -\gamma_2 + (\gamma - 1) \left( \phi_3 + \frac{a_1 \phi_5}{a_3} \right) \quad (14')$$

which equals (14) for  $\gamma = 1$ . In the case of a  $y^f$  shock, intervention in the foreign exchange market is required to keep the exchange rate constant both because the optimal policy displaces  $r$  and because of the direct effect of  $y^f$  on the current account. With different amounts of intervention in the case of incomplete sterilization, the money supply response via  $\lambda$  which maintains the initial level of aggregate demand for  $y^f$  and IS shocks differs.

With the  $\lambda$  in (14') for  $0 \leq \gamma \leq 1$ , the optimal policy fixes  $x$ , and has  $r$  respond to  $y^f$ , given  $p$  and  $y$ , so that the aggregate demand curve is not displaced, while the money market clears. The policy also stabilizes  $(y - y^*)$ , the overall price index and the terms of trade.

### B. LM Shock ( $v_3$ )

If the only disturbances to the system stem from the LM equation, the optimal policy setting is,

$$\lambda \rightarrow \infty \quad \mu \neq -(\phi_1 + \phi_4) \quad (15)$$

which sets  $\psi_{16} = \psi_{26} = 0$ . This is a generalized form of Poole's well known result. The policy also stabilizes employment,  $(y - y^*)$  and the general price index. The condition on  $\mu$  guarantees that an equilibrium exchange rate exists when the interest rate is targeted.

The optimal policy in (15) holds for all allowable values of  $b$ . Since the disturbance in the money market is not allowed to affect  $r$ , nothing else in the system is altered, except for the money stock, which simply satisfies the demand for money at the fixed  $r$ . Therefore, the value of  $b$  is irrelevant, since the exchange rate is not affected by this disturbance and thus cannot displace the aggregate supply curve.

### C. Balance of Payments Shock ( $v_4$ )

A rise in  $v_4$  increases the balance of payments surplus, *ceteris paribus*. Since the domestic bond market is the excluded market, a positive  $\Delta v_4$  implies a shift from foreign bonds or currency into domestic bonds.<sup>13</sup> If this is the only disturbance to the system, two policy settings exist which satisfy all objectives. These are,

$$\left\{ \begin{array}{l} \mu \rightarrow \infty \\ \gamma = 1, \\ \lambda \neq -\gamma_2 \end{array} \right. \quad \text{or,} \quad \left\{ \begin{array}{l} \mu \rightarrow \infty \\ \lambda \rightarrow \infty \\ 0 \leq \gamma \leq 1 \end{array} \right. \quad (16)$$

Mathematically, (16) works by setting  $\alpha_{18} = \alpha_{28} = 0$ , which sets  $\psi_{17} = \psi_{27} = 0$ .

These policies work as follows. A positive  $\Delta v_4$  would drive down  $x$ , but since  $\mu \rightarrow \infty$  the exchange rate is fixed at  $x = \bar{x}$ . To keep  $x$  unchanged in the face of an increase in the balance of payments surplus, the monetary authority must purchase foreign exchange reserves which will increase the money stock driving  $r$  down, thus displacing the aggregate demand schedule. To prevent this, sterilization could be complete, i.e.,  $\gamma$  could be set to one; or, the interest rate could be fixed at  $\bar{r}$ , with  $0 \leq \gamma \leq 1$ . Either version of (16) implies that the increased demand for domestic bonds is satisfied at  $r = \bar{r}$  by an increased bond supply. The condition that  $\lambda \neq -\gamma_2$

---

13. A  $v_4$  shock together with another shock would represent a balance of payments shock from sources other than this type of portfolio shift. A shift from domestic money to foreign bonds would, for example, be a negative  $v_3$  shift and a negative  $v_4$  shift.

in (16) guarantees that a unique equilibrium interest rate exists.

#### D. Aggregate Supply Shock ( $v_2$ )

In the event of a  $v_2$  disturbance, it is not possible to satisfy all targets with a given policy setting. Policy settings do exist which satisfy each of the objectives of interest separately. These policy settings, however, generally differ from one target to another.

A joint setting for  $\mu$  and  $\lambda$  which stabilizes the domestic and overall price levels ( $p$  and  $\bar{p}$ ), as well as the terms of trade ( $\varepsilon$ ) is

$$\mu \rightarrow \infty \quad \lambda = -(\gamma_2 + a_1\gamma_1) + (\gamma - 1) [\phi_3 + a_1\phi_2] \quad (17)$$

with no constraints on the sterilization coefficient,  $\gamma$ . This policy setting fixes  $x$  by setting  $\mu \rightarrow \infty$ . In addition,  $\lambda$  is set so that the *ceteris paribus* sensitivity of  $r$  to  $y$  ( $\alpha_{11}$  in (9)) is such that the slope of the aggregate demand schedule will be horizontal (i.e.  $\partial p / \partial y$  from (1), with (9) inserted and  $x = \bar{x}$ , equals zero). This stabilizes  $p$  and with  $x$  fixed stabilizes  $i$  and  $\varepsilon$  as well.

The policy given by (17) does not stabilize  $y$  or  $(y - y^*)$ . Policy settings can be derived which meet each of these objectives, nonoverlapping settings, however. The ranking of these policies relative to the policy in (17) which achieves price stability would depend on the relative weights which the policymaker attaches to price versus output stability.<sup>14</sup>

#### E. Foreign Price Shock ( $p^f$ )

If  $\Delta p^f$  is the only disturbance to the system, there exists a policy setting which satisfies all objectives. This optimal policy setting is,

---

14. Were we to further consider individual goals, wage indexation would become important to our analysis. Were we to look at policies which stabilize output around full-information output, for example, a number of issues arise concerning the advantage of using wage indexation in the face of supply shocks which (in some models) leaves monetary policy free to deal with other shocks. On this issue see Fethke and Jackman [1984], Devereux [1988] and Van Hoose and Waller [1989]. It is the case in our model, however, that even if wage indexation is used in concert with monetary policy, it is not possible to satisfy all five stabilization goals or even to stabilize price and output in the presence of a supply shock.

$$\lambda \rightarrow \infty \quad 0 \leq \gamma \leq 1 \quad \mu = -\phi_4 \quad (18)$$

This policy sets  $\alpha_{14} = 0$  and  $\alpha_{24} = -1$ , which implies that  $\psi_{12} = \psi_{22} = 0$ . This works for  $0 \leq b \leq 1$  in the following way. Setting  $\alpha_{24} = -1$  means that  $\mu$  is set so that any change in foreign price is exactly offset by an exchange rate change, so that  $p^f + x$  is unaffected. This implies no effect, *ceteris paribus*, on domestic spending or aggregate supply. The change in  $x$  will, however, disturb  $r$  and thus aggregate demand, hence  $\lambda \rightarrow \infty$  which fixes  $r$ . Thus, the policy setting in (18) completely insulates the domestic economy from a foreign price shock.<sup>15</sup>

### F. Foreign Interest Rate Shock ( $r^f$ )

If the only disturbance to the system is a change in the foreign interest rate, the optimal policy setting is

$$\lambda \rightarrow \infty \quad \mu \rightarrow \infty \quad 0 \leq \gamma \leq 1 \quad (19)$$

which can be seen to set  $\psi_{13} = \psi_{23} = 0$ , for  $0 \leq b \leq 1$ . This is identical to one of the policies in (16), the optimal policy setting for a balance of payments shock. Notice that the other optimal policy in (16),  $\mu \rightarrow \infty$ ,  $\lambda \neq -\gamma_2$  and  $\gamma = 1$  will not work here because while  $\Delta v_4$  does not directly affect the demand for money,  $\Delta r^f$  does; hence  $\Delta r^f$  will disturb the interest rate, at a given  $p$  and  $y$ , even with  $x$  fixed and with no change in the money stock due to complete sterilization.

## VI. Summary of Results

Our optimal policy settings are gathered together in Table 1. All of the policy settings in Table 1 simultaneously satisfy our five policy objectives. There is no policy setting that can do this in response to a supply disturbance. The setting in (17) simultaneously stabilizes domestic price, the overall price index, and the terms of trade, but not output or output around full-information output in the case of

---

15. As noted in footnote 6, had we used the general price index,  $\bar{p}$ , instead of  $p$  in the money demand function, the foreign price level would have appeared there. Because the setting in (18) targets the interest rate, this alternative formulation would yield the same optimal setting as does (18).

supply shock.

Several propositions emerge from these results.

1. Targeting the interest rate and fixing the exchange rate ( $\lambda \rightarrow \infty$  and  $\mu \rightarrow \infty$ ) is optimal in the face of three disturbances occurring simultaneously: an LM shock ( $v_3$ ), a balance of payment (capital flow,  $v_4$ ) shock, and a foreign interest rate ( $r^f$ ) shock.

**Table 1**  
**Policy Settings Which Satisfy All Six Policy Objectives**  
**(No Restriction on Wage Indexation)**

Policy Setting		$\lambda$	$\mu$	$\gamma$
Shocks				
IS	$(v_1)$	$-\gamma_2 + \phi_3(\gamma - 1)$	$\infty$	free
Foreign Output	$(y^f)$	$-\gamma_2 + \left(\frac{a_1\phi_3}{a_3} + \phi_3\right)(\gamma - 1)$	$\infty$	free
Supply <sup>1</sup>	$(v_2)$			
LM	$(v_3)$	$\infty$	arbitrary but $\neq -(\phi_1 + \phi_4)$	free
Balance of Payments	$(v_4)$	arbitrary but $\neq -\gamma_2$	$\infty$	$\gamma=1$
		$\infty$	$\infty$	free
Foreign Price	$(p^f)$	$\infty$	$-\phi_4$	free
Foreign Interest Rate	$(r^f)$	$\infty$	$\infty$	free

Note: 1 See(17) for a policy setting that stabilizes only the terms of trade, domestic price and the overall price index.

2. If there is complete sterilization, then a fixed exchange rate ( $\mu \rightarrow \infty$ ) and a vertical LM schedule ( $\lambda = -\gamma_2$ ) will be optimal in the face of both domestic IS shocks and shocks to foreign output ( $y^f$ ). Without perfect sterilization, the optimal responses to these shocks require different settings of  $\lambda$ , though both include a fixed exchange rate.

3. Taking (1) and (2) together we see that a fixed exchange rate is part of the optimal policy setting for all the financial shocks ( $v_3, v_4, r^f$ ), as well as the domestic IS shocks and foreign output shocks, i.e., for *all* shocks that do not directly shift the domestic aggregate supply curve. This suggests that if these types of shocks predominate, a desirable policy mix might be a fixed exchange rate (*hard target zone*), together with a degree of interest rate smoothing that depends on the relative importance of the financial versus the goods market disturbances. Put differently, *fixing the exchange rate reduces the optimal policy problem to the original Poole problem, as long as the supply curve is not disturbed* (no  $v_2$  or  $p^f$  shocks).<sup>16</sup> Notice also that given a  $v_2$  shock a fixed exchange rate is a part of the policy that achieves three of the stabilization goals.

4. The optimal policy in the presence of a foreign price shock does not include a fixed exchange rate. Instead, via intervention in the foreign exchange market, the exchange rate is forced to adjust such that the effect of the foreign price shock on the terms of trade is just offset. The interest rate must be pegged, as with financial shocks, in this case to prevent the adjustment of the exchange rate from disturbing equilibrium in the money market.

Except in the case of shocks that directly shift the aggregate supply function, our results favor a fixed exchange rate system, or alternatively a target zone approach,

---

16. As in Poole's analysis, the fact that the optimal policy setting varies depending on the relevant configuration of shocks, means that to be operational this policy framework requires knowledge of which shocks are predominant at a given time. This is a limitation because there will no doubt be times when policymakers view shocks as more or less equally likely to originate from many sources. Still, there are times when the monetary authority can discern the predominant source of shocks. In the United States, such periods can be argued to include 1974-75, 1979, 1982 and in late 1990. On the 1979 period, for example, see the discussion in Wallich [1980]; Wallich, at the time a Federal Reserve Board Governor, explicitly recognized the original Poole insight by stating that "policymakers must take into account at all times the fact that both the economy and the demand for money may exhibit instability. The choice of a money control strategy depends upon which of the two instabilities appears to predominate at any particular time" (Wallich 1980, p. 51).

rather than a flexible exchange rate.<sup>17</sup> It should be kept in mind, however, that our model is one of imperfect capital mobility, which allows the policymaker to affect the exchange rate by sterilized intervention. Also, our results are predicated on the assumption that countries have adequate foreign exchange reserves to carry out the necessary foreign exchange market intervention.<sup>18</sup> Finally, our results favor a fixed exchange rate (in the absence of shocks that shift the domestic supply schedule) as part of an optimal policy package. They do not indicate a ranking of exchange rate regimes *per se*.<sup>19</sup>

## Appendix A

This appendix explains the derivation of equation (2), the model's aggregate supply schedule.

We begin with the production function, assumed to be Cobb-Douglas

$$Y = K^c L^{1-c} V_2 \quad (\text{A1})$$

where  $Y$  = real output,  $K$  and  $L$  are capital and labor inputs and  $V_2$  is a productivity shock (all in levels not logarithms). Setting the marginal product of labor equal to the real wage gives the following demand for labor equation

$$l^d = \ln(1-c) + y - w + p \quad (\text{A2})$$

where  $l^d$  = the logarithm of labor demand, and  $w$  = the logarithm of the nominal wage rate. Setting  $K^c = 1$ , taking the logarithm of equation (A1) ( $y_1 = (1-c)l + v_2$ ;

17. A fixed exchange rate is also part of the policy (equation 17) which stabilizes three goals in the presence of a domestic supply shock ( $v_2$ )

18. Within our framework, foreign exchange reserves will follow a random walk. This implies that in the long-run a realization of the shocks will occur that leads to a balance of payments crisis. Such crises have received extensive attention in papers going back to Krugman [1979] and in the more recent literature on target zones. (See, for example, Krugman and Rotemberg [1992].) Our analysis considers only the short-run and assumes that reserves remain at levels that prevent *speculative attacks*. We are indebted to a referee on this point.

19. Our results are not, therefore, in direct conflict with earlier work in the Mundell-Flemming tradition which showed that flexible exchange rates provide a greater degree of insulation from foreign demand shocks. On the comparison of pure exchange rate policy regimes (rates fixed versus aggregates fixed) our results do not differ from Henderson's [1982].

where  $v_2$  is the log of  $V_2$ , then substituting (A2) for the log of labor demand in the result yields

$$y = \left(\frac{1-c}{c}\right) [\ln(1-c) - (w-p)] + \frac{1}{c} v_2 \quad (\text{A3})$$

Since workers consume both domestic and foreign goods, the supply of labor is assumed to be responsive to the nominal wage relative to the overall price index, i.e.,

$$l^s = n_0 + n(w - \bar{p}) \quad (\text{A4})$$

The contract wage,  $w_t^c$ , is set so that  $E_{t-1} l_t^d = E_{t-1} l_t^s$ , which yields

$$w_t^c = \frac{E_{t-1}(p_t + nc\bar{p}_t) + \ln(1-c) - n_0 c}{1+nc} \quad (\text{A5})$$

The actual wage,  $w_t^a$ , is taken to be indexed to the overall price via equation (8) in the text

$$w_t = w_t^c + b(\bar{p}_t - E_{t-1}\bar{p}_t) \quad (8)$$

Using (A5) in (8) and the result in (A3) yields the aggregate supply schedule in (2), where the  $c$ 's are defined in Appendix B (with  $c_0$  ignored since it is not used).

## Appendix B

This appendix gives the values of parameters from the text as follows:

<Equation 2>

$$c_1 = \frac{1-c}{c(1+nc)} \quad c_2 = \frac{(1-c)nc}{c(1+nc)} \quad c_3 = \frac{(1-c)}{c} \quad c_4 = \frac{1}{c}$$

<Equations 9 and 10>

$$\alpha_{11} = \frac{-\gamma_1(\mu + \phi_1 + \phi_4) + (\mu(\gamma - 1) - \gamma_3)\phi_2}{\Delta}$$

$$\alpha_{12} = \frac{-(\mu + \phi_1 + \phi_4) + \phi_1(\mu(\gamma - 1) - \gamma_3)}{\Delta}$$

$$\alpha_{13} = \frac{-[\mu(\gamma - 1) - \gamma_3]\phi_5}{\bar{\Delta}}$$

$$\alpha_{14} = \frac{-(\mu(\gamma - 1) - \gamma_3)\phi_1}{\bar{\Delta}}$$

$$\alpha_{15} = \alpha_{16} = \frac{\gamma_3(\mu + \phi_1 + \phi_4) + \phi_4(\mu(\gamma - 1) - \gamma_3)}{\bar{\Delta}}$$

$$\alpha_{17} = \frac{-(\mu + \phi_1 + \phi_4)}{\bar{\Delta}}$$

$$\alpha_{18} = \frac{-[\mu(\gamma - 1)\gamma_3]}{\bar{\Delta}}$$

$$\alpha_{21} = \frac{-(\lambda + \gamma_2)\phi_2 + \phi_3\gamma_1}{\bar{\Delta}}$$

$$\alpha_{22} = \frac{-(\lambda + \gamma_2)\phi_1 + \phi_3}{\bar{\Delta}}$$

$$\alpha_{23} = \frac{(\lambda + \gamma_2)\phi_5}{\bar{\Delta}}$$

$$\alpha_{24} = \frac{(\lambda + \gamma_2)\phi_1}{\bar{\Delta}}$$

$$\alpha_{25} = \alpha_{26} = \frac{-\phi_4(\lambda + \gamma_2) - \phi_3\gamma_3}{\bar{\Delta}}$$

$$\alpha_{27} = \frac{\phi_3}{\bar{\Delta}}$$

$$\alpha_{28} = \frac{\lambda + \gamma_2}{\bar{\Delta}}$$

where  $\bar{\Delta} = \phi_3(\mu(\gamma - 1) - \gamma_3) - (\lambda + \gamma_2)(\mu + \phi_1 + \phi_4)$ .

<Equations 11 and 12>

$$\psi_{11} = \frac{HC - GF}{\Delta^*}$$

$$\psi_{12} = \frac{KC - JF}{\Delta^*}$$

$$\psi_{21} = \frac{AH - DG}{\Delta^*}$$

$$\psi_{22} = \frac{AK - DJ}{\Delta^*}$$

$$\psi_{13} = \frac{CM - LF}{\Delta^*}$$

$$\psi_{14} = -F / \Delta^*$$

$$\psi_{15} = \frac{C}{\Delta^*}$$

$$\psi_{16} = \frac{CQ - NF}{\Delta^*}$$

$$\psi_{17} = \frac{CS - RF}{\Delta^*}$$

$$\psi_{23} = \frac{AM - DL}{\Delta^*}$$

$$\psi_{24} = -D / \Delta^*$$

$$\psi_{25} = \frac{A}{\Delta^*}$$

$$\psi_{26} = \frac{AQ - DN}{\Delta^*}$$

$$\psi_{27} = \frac{AS - DR}{\Delta^*}$$

where

$$\Delta^* = DC - AF$$

$$A = 1 + a_1\alpha_{11} - a_2\alpha_{21}$$

$$C = a_2\alpha_{22} - a_2 - a_1\alpha_{12}$$

$$D = 1 + (1-a)c_3b\alpha_{21}$$

$$F = c_1 + c_2 - c_3b(a + (1-a)\alpha_{22})$$

$$G = a_3 - a_1\alpha_{13} + a_2\alpha_{23}$$

$$H = -c_3b(1-a)\alpha_{23}$$

$$J = a_2 - a_1\alpha_{14} + a_2\alpha_{24}$$

$$K = -c_3b(1-a)(1 + \alpha_{24})$$

$$L = a_2\alpha_{26} - a_1\alpha_{16}$$

$$M = -c_3b(1-a)\alpha_{26}$$

$$N = a_2\alpha_{27} - a_1\alpha_{17}$$

$$Q = -c_3b(1-a)\alpha_{27}$$

$$R = a_2\alpha_{28} - a_1\alpha_{18}$$

$$S = -c_3(1-a)\alpha_{28}$$

<Equations 13>

$$\beta_1 = \frac{1+nca}{1+nc}$$

$$\beta_2 = \frac{cn(1-a)}{1+nc}$$

$$\beta_3 = \frac{1}{1+nc}$$

## References

- Aizenman, Joshua [1985], "Wage Flexibility and Openness," *Quarterly Journal of Economics*, 100, pp. 539-50.
- Aizenman, Joshua and Jacob Frenkel [1985], "Optimal Wage Indexation, Foreign

- Exchange Intervention, and Monetary Policy," *American Economic Review*, 75, pp. 402-23.
- [1986], "Supply Shocks, Wage Indexation, and Monetary Accommodation," *Journal of Money, Credit and Banking*, 18, pp. 304-22.
- Artis, M.J. and David A. Currie [1981], "Monetary Targets and the Exchange Rate: A Case for Conditional Targets," *Oxford Economic Papers*, 33, Supplement, pp. 176-200.
- Barro, Robert J. and David Gordon [1983], "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 91, pp. 589-610.
- Benavie, Arthur [1983], "Achieving External and Internal Targets with Exchange-Rate and Interest-Rate Intervention," *Journal of International Money and Finance*, 2, pp. 75-85.
- Benavie, Arthur and Richard Froyen [1983], "Combination Monetary Policies to Stabilize Price and Output under Rational Expectations," *Journal of Money, Credit and Banking*, 15, pp. 186-198.
- Boyer, Russell S. [1978], "Optimal Foreign Exchange Market Intervention," *Journal of Political Economy*, 86, pp. 1045-55.
- Bradley, Michael D. and Dennis W. Jansen [1988], "Price Rules, Wage Indexation and Optimal Monetary Policy," *Journal of Macroeconomics*, 10, pp. 515-38.
- Canzoneri, Matthew B., Dale W. Henderson and Kenneth Rogoff [1983], "The Information Content of the Interest Rate and Monetary Policy," *Quarterly Journal of Economics*, 98, pp. 545-66.
- Cox, W. Michael [1980], "Unanticipated Money, Output and Prices in the Small Economy," *Journal of Monetary Economics*, 6, pp. 359-84.
- Craine, Roger and Arthur, M. Havenner [1981], "Choosing a Monetary Instrument: The Case of Supply-Side Shocks," *Journal of Economic Dynamics and Control*, 3, pp. 217-34.
- Devereux, Michael [1988], "The Optimal Mix of Wage Indexation and Foreign Exchange Market Intervention," *Journal of Money, Credit and Banking*, 20, pp. 381-92.
- [1989], "A Positive Theory of Inflation and Inflation Variance," *Economic Inquiry*, 27, pp. 105-16.
- Fethke, Gary and Richard Jackman [1984], "Optimal Monetary Policy, Endogenous Supply and Rational Expectations," *Journal of Monetary Economics*, 13, pp. 211-24.

- Flood, Robert P. and Nancy P. Marion [1982], "The Transmission of Disturbances under Alternative Exchange-Rate Regimes with Optimal Indexation," *Quarterly Journal of Economics*, 97, pp. 43-66.
- Friedman, Benjamin M. [1975], "Targets Instruments and Indicators of Monetary Policy," *Journal of Monetary Economics*, 1, pp. 443-73.
- Frenkel, Jacob A. and Joshua Aizenman [1982], "Aspects of the Optimal Management of Exchange Rates," *Journal of International Economics*, 13, pp. 231-56.
- Henderson, Dale W. [1979], "Financial Policies in Open Economies," *American Economic Review*, 69, pp. 232-39.
- [1984], "Exchange Market Intervention Operations: Their Effects and Their Role in Financial Policy," in *Exchange Rate Theory and Practice*, edited by John F.O. Bilson and Richard C. Marston, pp. 357-406. Chicago: University of Chicago Press, 1984.
- Krugman, Paul W. [1979], "A Model of Balance of Payments Crisis," *Journal of Money, Credit and Banking*, 11, pp. 311-25.
- [1990], "Equilibrium Exchange Rates," in *International Policy Coordination and Exchange Rate Fluctuations*, edited by William H. Branson, Jacob A. Frenkel and Morris Goldstein, pp. 159-87. Chicago: University of Chicago Press.
- Krugman, Paul W. and Julio Rotemberg [1992], "Speculative Attacks on Target Zones," in *Exchange Rate Targets and Currency Bonds*, edited by Paul Krugman and Marcus Miller, Cambridge: Cambridge University Press.
- LeRoy, S.F. and Roger N. Waud [1977], "Application of the Kalman Filter in Short-Run Monetary Control," *International Economic Review*, 18, pp. 195-207.
- Marston, Richard C. [1982], "Wages, Relative Prices, and the Choice Between Fixed and Flexible Exchange Rates," *Canadian Journal of Economics*, 15, pp. 87-103.
- [1985], "Stabilization Policies in Open Economies," in *Handbook of International Economics*, edited by Ronald W. Jones and Peter B. Kenen, pp. 859-916, Amsterdam: North Holland.
- [1984], "Real Wages and the Terms of Trade: Alternative Indexation Rules for an Open Economy," *Journal of Money, Credit, and Banking*, 16, pp. 285-301.
- and Stephen J. Turnovsky [1985], "Imported Materials Prices, Wage Policy

- and Macroeconomic Stabilization," *Canadian Journal of Economics*, 18, pp. 273-84.
- Parkin, Michael [1978], "A Comparison of Alternative Techniques of Monetary Control under Rational Expectations," *The Manchester School*, 46, pp. 252-86.
- Poole, William [1970], "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model," *Quarterly Journal of Economics*, 84, pp. 197-216.
- Roper, Don E. and Stephen J. Turnovsky [1980], "Optimal Exchange Market Intervention in a Simple Stochastic Macro Model," *Canadian Journal of Economics*, 13, pp. 296-309.
- Turnovsky, Stephen J. [1976], "The Relative Stability of Alternative Exchange Rate Systems in the Presence of Random Disturbances," *Journal of Money, Credit, and Banking*, pp. 29-50.
- [1987], "Optimal Monetary Policy and Wage Indexation under Alternative Disturbances and Information Structures," *Journal of Money, Credit, and Banking*, 19, pp. 157-80.
- [1984], "Exchange Market Intervention Under Alternative Forms of Exogenous Disturbances," *Journal of International Economics*, 17, pp. 279-97.
- [1983], "Wage Indexation and Exchange Market Intervention in a Small Open Economy," *Canadian Journal of Economics*, 16, pp. 574-92.
- [1980], "The Choice of Monetary Instruments under Alternative Forms of Price Expectations," *The Manchester School*, 48, pp. 39-62.
- Van Hoose, David D. and Christopher J. Waller [1989], "Islands, Indexation and Monetary Policy," *Economic Inquiry*, 27, pp. 705-17.
- and ——— [1991], "Discretion, Wage Indexation and Inflation," *Southern Economic Journal*, 58, pp. 356-67.
- [1992], "Discretionary Monetary Policy and Socially Efficient Wage Indexation," *Quarterly Journal of Economics*, 107, forthcoming.
- Wallich, Henry C. [1980], "A New Strategy for the Federal Reserve," *Challenge Magazine*, 22, pp. 49-51.
- Woglom, Geoffrey [1979], "Rational Expectations and Monetary Policy in a Simple Macroeconomic Model," *Quarterly Journal of Economics*, 93, pp. 91-105.