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Commercial Policy with Vertical Product Differentiation

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Abstract

We examine the nature of commercial and domestic policy (tariffs, taxes/subsidies, and quality restrictions) in a model of vertical product differentiation. A foreign firm competes with a domestic firm in the latter's market, producing products of varying quality, and competing in prices. We show that a specific tariff on the foreign firm raises overall welfare in the domestic economy, while an ad valorem tariff has a similar effect only when the foreign firm produces the lower quality product. Tariffs on the foreign firm typically induce the domestic firm to upgrade the quality of its product, when it produces the lower quality product. A subsidy is always the optimal policy towards the domestic firm. If quality restrictions are imposed on the foreign firm, the domestic firm upgrades quality, and overall welfare in the domestic economy is once again higher. (JEL: F12, F13)

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I. Introduction

Theoretical analysis of commercial policy has gone through a significant change in recent years. Models of international trade have incorporated imperfect competition as well as differentiated goods, so that tariffs, quotas, and other restrictions have been shown in some cases to have beneficial strategic effects ⟨see Krugman [1984], Brander and Spencer [1983, 1985], Dixit and Kyle [1985], Dixit and Grossman [1986], and Eaton and Grossman [1986]⟩.

This line of research has been extended to consider the impact of trade policy on quality-differentiated goods as well ⟨see, for example, Falvey [1979], Donnenfeld [1986, 1988], Bond [1988], Das and Donnenfeld [1987, 1989], and Krishna [1987]⟩.

In particular Das and Donnenfeld [1989] examine the question of endogenous quality and commercial policy. Their model focuses on the effect of quotas and quality restrictions when firms compete in quantities. They find that a quota enhances the profits of the domestic firm and leads it to improve quality, if it produces the lower quality product. Furthermore, they show that quotas have an ambiguous effect on national welfare, while minimum quality standards lower domestic profits and welfare.

In a recent paper in this journal, Wall [1994] has shown the effect of specific and ad valorem tariffs on quality-differentiated goods. Wall assumes qualities are fixed and that foreign and domestic firms are perfect competitors. Hence prices are equal to the constant marginal cost of each product variety.

In the present paper we extend the theory of commercial policy to quality-differentiated goods when qualities are endogenously determined. Furthermore we consider the case of two firms, a domestic firm and a foreign firm producing goods of different qualities, competing in the domestic market. Unlike Das and Donnenfeld [1989], we focus on price competition between the firms and assume that the firms determine quality before they engage in price competition.[1] Apart from analyzing specific and ad valorem

1. In Das and Donnenfeld [1989] quantity and quality are simultaneously chosen. We believe that it is more realistic to assume that firms set quality before they engage in

tariffs, we examine the effect of domestic policy (taxes/subsidies) on the domestic firm, an issue not addressed in either Das and Donnenfeld [1989] and Wall [1994]. The results we obtain differ significantly from the existing literature.

Here are some of our main findings:

(i) The effect of a specific tariff on the foreign firm depends on whether it produces the higher or lower quality product. The domestic firm upgrades quality if it produces the lower quality product, and downgrades quality if it produces the higher quality product. However domestic profits and overall national welfare are always higher. When an ad valorem tariff is used instead, the domestic firm downgrades quality if it produces the higher quality product, but the effect on quality is ambiguous when it produces the lower quality product.

(ii) A specific subsidy (tax) on the domestic firm leads to a(n) decrease (increase) in its quality, higher (lower) profits, and higher (lower) overall national welfare. The results are generally the same with an ad valorem subsidy (tax), except that the domestic firm increases (decreases) its quality when it produces the lower (higher) quality product.

(iii) In response to minimum quality standards on the foreign firm, the domestic firm always upgrades quality, and overall national welfare is higher.

The game between the government and the firms is assumed to be played in 3 stages. The government decides between imposing tariffs on the foreign firm or taxes/subsidies on the domestic firm in stage 1. In stage 2, the firms choose their optimal quality in stage 2, and in stage 3 they maximize profits by choosing appropriate prices.

In section II we present our assumptions, and analyze the impact of a specific tariff on the foreign firm and a specific tax/subsidy on the domestic firm. Section III examines the case of an ad valorem tariff on the foreign firm and an ad valorem tax/subsidy on the domestic firm, while section IV examines the effects of minimum quality standards (MQS) imposed on the

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either price, or quantity competition. In fact Das and Donnenfeld mention, "The extent to which the sequence of decision-making is reasonable depends on the nature of the product. For example, for products such as automobiles, capacity and quality determination requires lengthy planning of plants, specifications, body hardware, etc. and hence are likely to precede price determination."

foreign firm. Conclusions are given in section V.

II. Specific Tariffs and Taxes

The model we present here is similar to Mussa and Rosen [1978]. The other seminal papers in the industrial organization literature are Spence [1976], and Shaked and Sutton [1982]. Two firms, one foreign and the other domestic, compete in prices in the domestic market. One firm produces the higher quality product, denoted by subscript 1, while the other firm produces the lower quality product, denoted by subscript 2. Thus Firm 1 produces the product q_1, the quality of which is k_1, and charges price p_1. Firm 2 produces q_2 whose quality is k_2, and charges p_2. Either firm could be the foreign and/or the domestic firm. We assume $1 \geq k_1 > k_2 > 0$.[2] Consumers in the domestic market are uniformly distributed in the interval [0,1] according to their tastes. The taste parameter is denoted by θ. If the prices were equal, all the consumers would prefer the higher quality product. We therefore assume that $p_1 > p_2$. The utility function of the consumers is given by $u(0) = I$, and $u(k_i) = \theta k_i + I - p_i$, where i stands for product 1 or 2, and I is the income of the consumer. We assume that every consumer buys either 0 or 1 unit of one, and only one, of the products. The above utility function is separable in k_i and p_i. In fact, the utility function we use is the net consumer surplus function.[3]

The model we analyze has three stages. In stage 1, the government decides whether to impose a tax (or a subsidy) on the quantity produced by one of the firms in the domestic market. In stage 2, after the policy of the government is revealed, each firm chooses its optimal quality k_i. In stage 3, the firms maximize their profits by choosing the appropriate prices p_i. To find the equilibrium prices, qualities, and trade policy, we work backward

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2. When $k_1=k_2$, both products are homogeneous, and therefore their prices should be equal. If prices are not equal, all consumers will buy the product from the lower priced firm. Thus both firms will drive prices down till profits are zero, and neither firm has any incentive to remain in the market. Since the competition is in prices, k_1 cannot equal k_2, and without any loss in generality we assume $k_1 > k_2$.
3. The term $\theta k_i + I$ is the gross consumer surplus, and by subtracting the expenditure p_i from it, we get the net consumer surplus.

from stage 3 to stage 1. Eaton and Grossman [1986] have shown that the optimal trade policy for the domestic government is to impose a tax on the quantity sold by the firms, if the firms play a Bertrand game. In their 2-stage model they assume that both firms compete in a third market, and thus the welfare of domestic consumers is ignored. In our 3-stage model however, the products are sold in the domestic market, and hence the domestic government can no longer ignore the welfare of consumers. It is now no longer obvious that a tax is optimal, even if the commodities are sold in a foreign market.

We write the cost functions as follows,[4]

$$c_1(k_1) = 0.5k_1q_1 + k_1^2 \tag{1}$$

$$c_2(k_2) = 0.5k_2q_2 + k_2^2 \tag{2}$$

The cost functions show that the marginal cost increases as quality increases (since $k_1 > k_2$), and in stage 2, when quality is being determined, the costs are quadratic.[5]

We derive the demand functions that the firms face in the following way. There exists a consumer whose taste parameter is given by θ_h and who is indifferent between buying products 1 and 2. We find θ_h by equating $u_h(k_1)$ and $u_h(k_2)$. Since,

$$u_h(k_1) = \theta_h k_1 + I - p_1, \tag{3}$$

$$u_h(k_2) = \theta_h k_2 + I - p_2, \tag{4}$$

Since we require $u_h(k_1) = u_h(k_2)$, we have,

$$\theta_h k_1 + I - p_1 = q_h k_2 + I - p_2 \tag{5}$$

Simplifying, we get,

$$\theta_h = \frac{p_1 - p_2}{k_1 - k_2} \tag{6}$$

4. We have chosen the value 0.5 for the cost function coefficient for analytical simplicity. Note that the results in the model hold for all values in the interval [0,1].

5. Motta [1993] also considers endogenous quality choice under price competition, but first assumes zero marginal costs at the quality choosing stage, and later assumes zero marginal costs at the price-setting stage. We believe our version of the cost function to be more general.

All consumers whose taste parameter $\theta > \theta_h$ will buy from Firm 1. Therefore the demand function that Firm 1 faces can be written as,[6]

$$q_1 = 1 - \frac{p_1 - p_2}{k_1 - k_2} \tag{7}$$

Similarly, there exists a consumer whose taste parameter is given by θ_i and who is indifferent between buying q_2 and not buying anything at all. We derive the value of θ_i by equating $u_i(k_2) = \theta_i k_2 + I - p_2$ with $u_i(0) = I$. Therefore,

$$\theta_i = \frac{p_2}{k_2} \tag{8}$$

A consumer whose taste parameter is lower than θ_i buys neither product. A consumer whose θ lies between θ_h and θ_i buys the lower quality product q_2 from Firm 2. Therefore the demand function of Firm 2 is given by,[7]

$$q_2 = \frac{p_1 - p_2}{k_1 - k_2} - \frac{p_2}{k_2}^{8} \tag{9}$$

A. Tariff/Tax on the Firm Producing the High Quality Product

The firms in stage 3 face linear demand functions in prices. The imposition of a tax 'softens' competition when prices are strategic complements, and prices of both products increase as a result of a tariff.

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6. Note that we have dropped the number of consumers from the demand function for simplicity.
7. Note the demand function of q_2 can be written as $q_2 = \frac{p_1 k_2 - p_2 k_1}{k_2(k_1 - k_2)}$. In order for the firm producing the lower quality product to serve some portion of the market, the quantity q_2 has to be positive. Therefore, $p_1 k_2 - p_2 k_1$ should be greater than 0. By rearranging the above expression we get $\frac{p_1}{k_1} > \frac{p_2}{k_2}$. The above inequality tells us the price of one unit of quality of the high quality product should be greater than that of the lower quality product.
8. When $\theta_h > 1$, $q_1 = 0$, and when $\theta_h < q_i$, $q_2 = 0$. Therefore θ_h must be > than θ_i to ensure well defined demand functions. Furthermore if p_1/k_1 is > 1, the demand for product 1 is 0, since consumers will derive negative consumer surplus from its consumption. In order to have both firms serving the market we require that $1 > \frac{p_1}{k_1} > \frac{p_2}{k_2} > 0$. Note that the Nash equilibrium prices in equations (14) and (15) do satisfy the above inequality as long as $1 > k_1 > k_2 > d > 0$.

We consider two situations with respect to optimal policy.

(a) The foreign firm produces the higher quality product and the government imposes a tariff on the foreign firm.

(b) The domestic firm produces the higher quality product and the government imposes a tax on the domestic firm

If $t > (<) 0$, then t can be viewed as a tariff (subsidy). To analyze the welfare effects of the policy we need to examine its impact on the government budget, the profits of the domestic firm, and consumers' surplus.

The demand functions are given in equations (7) and (9). In stage 3, the profit functions of Firm 1 and Firm 2 with a tariff/tax on Firm 1 are,

$$\Pi_1 = p_1 q_1 - 0.5k_1 q_1 - tq_1 - k_1^2 \tag{10}$$

$$\Pi_2 = p_2 q_2 - 0.5k_2 q_2 - k_2^2 \tag{11}$$

The first derivative of these profit functions with the respect to own prices is given by,

$$\frac{\partial \Pi_1}{\partial p_1} = 1 + \frac{0.5k_1 - 2p_1 + p_2 + t}{k_1 - k_2} \tag{12}$$

$$\frac{\partial \Pi_2}{dp_2} = \frac{(p_1 - p_2)k_2 - p_2 k_1 - p_2(k_1 - k_2) - 0.5k_1 k_2}{k_2(k_1 - k_2)} \tag{13}$$

By equating the above equations to 0 and solving for p_1 and p_2 we get,

$$p_1 = \frac{3(k_1^2 - 0.5k_1 k_2 + 0.67k_1 t)}{4k_1 - k_2} \tag{14}$$

$$p_2 = \frac{2.5(k_1 k_2 - 0.4k_2^2 + 0.4k_2 t)}{4k_1 - k_2} \tag{15}$$

The equilibrium quantities are given by,

$$q_1 = \frac{(2k_2 - 3k_1^2 - 0.5k_1 k_2 - t)}{-4k_1 + k_2} \tag{16}$$

$$q_2 = \frac{(2k_2^2 - 2.5k_1 k_2 - t)}{-4k_1 + k_2} \tag{17}$$

Prices in this model are strategic complements.[9] This implies that the reaction curves $r_1(p_2)$ and $r_2(p_1)$ slope upward.

The consumer surplus is defined as follows,

$$CS = \int_{\theta_i}^{\theta_h} (\theta_i k_2 - p_2)dq + \int_{\theta_h}^{1} (\theta k_1 - p_1)dq =$$

$$\frac{0.03125(k_1^2 - k_1k_2 - 2k_1t + 2k_2t)(k_1^3 - k_1^2k_2 - 2k_1^2t + k_1k_2t}{(k_1^2 - 1.25k_1k_2 + 0.25k_2^2)^2}$$

$$+\frac{0.03125(k_1^2 - k_1k_2 - 2k_1t + 2k_2t)(k_1^3k_2 - 1.25k_1^2k_2^2 + 0.25k_1k_2^3 + 2k_1^2k_2t - 0.5k_1k_2^2t)}{(k_1^5 - 2.75k_1^4k_2 + 2.6875k_1^3k_2^2 - 1.14k_1^2k_2^3 + 0.21875k_1k_2^4 - 0.015625k_2^5)}$$

$$+\frac{0.078125(k_1^3k_2 - 1.25k_1^2k_2^2 + 0.25k_1k_2^3 + 2k_12k_2t - 0.5k_1k_2^2t)^2}{(k_1^6k_2 - 3k_1^5k_2^2 + 3.375k_1^4k_2^3 - 1.8125k_1^3k_2^4 + 0.5k_1^2k_2^5 - 0.07k_1k_2^6 + 0.004k_2^7)} \qquad (18)$$

The government revenue function is,

$$G = tq_1 \qquad (19)$$

We now have enough information to proceed to stage 2. We can write the profit functions by substituting the values of p_1 and p_2 obtained in stage 3 as follows,

$$\Pi_1 = \left[\frac{0.1875(k_1^2 - 0.5k_1k_2 + 0.67k_1t)(k_1^2 - k_1k_2 - 2k_1t + k_2t}{k_1^3 - 1.5k_1^2k_2 + 0.5625k_1k_2^2 - 0.0625k_2^3)}\right] \qquad (20)$$

$$-\left[\frac{0.5(k_1^2 - k_1k_2 - 2k_1t + k_2t)}{4k_1^2 - 5k_1k_2 + k_2^2}\right] - \left[\frac{4t(k_1^2 - k_1k_2 - 2k_1t + k_2t)}{4k_1^2 - 5k_1k_2 + k_2^2}\right] - k_1^2$$

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9. We borrow the terminology, strategic complements and strategic substitutes, from Bulow *et al.* [1985]. The prices are defined to be strategic complements (strategic substitutes) if the second order cross derivatives of the profit functions $\frac{\partial^2\Pi_i}{\partial p_i \partial p_j}$ are positive (negative). The second derivatives of the profit functions are given by $\frac{\partial^2\Pi_1}{\partial p_1^2} = \frac{-2}{k_1 - k_2} < 0$, $\frac{\partial^2\Pi_2}{\partial p_2^2} = \frac{-2k_1}{k_2(k_1 - k_2)} < 0$, $\frac{\partial^2\Pi_1}{\partial p_1 \partial p_2} = \frac{1}{k_1 - k_2} > 0$ and $\frac{\partial^2\Pi_2}{\partial p_2 \partial p_1} = \frac{1}{k_1 - k_2} > 0$. since by assumptions $k_1 > k_2$, the second order cross derivatives help us conclude that the prices are strategic complements. The stability conditions are also satisfied since we can show that $\frac{\partial^2\Pi_1}{\partial p_1^2}\frac{\partial^2\Pi_2}{\partial p_2^2} - \frac{\partial^2\Pi_1}{\partial p_1 \partial p_2}\frac{\partial^2\Pi_2}{\partial p_2 \partial p_1} = \frac{4k_1}{k_2(k_1 - k_2)^2} - \frac{1}{(k_1 - k_2)^2} = \frac{(4k_1 - k_2)}{k_2(k_1 - k_2)^2} > 0$. Therefore the reaction curves at the plane of p_1 and p_2 do not intersect "the wrong way".

$$\Pi_2 = \left[\frac{0.078125(k_1k_2 - 0.4k_2^2 + 0.4k_2t)(k_1^3k_2 - 1.25k_1^2k_2^2 + 0.25k_1k_2^3 + 2k_1^2k_2t - 0.5k_1k_2^2t}{k_1^4k_2 - 1.75k_1^3k_2^2 + 0.9375k_1^2k_2^3 - 0.203125k_1k_2^4 + 0.015625k_2^5}\right]$$

$$-\left[\frac{0.0625k_2(k_1^3k_2 - 1.25k_1^2k_2^2 + 0.25k_1k_2^3 + 2k_1^2k_2t - 0.5k^1k_2^2t)}{k_1^3k_2 - 1.5k_1^2k_2^2 + 0.5625k_1k_2^3 - 0.625k_2^4}\right] - k_2^2 \qquad (21)$$

We want to maximize these functions with respect to k_1 and k_2. The first derivative of Π_1 is given by,

$$\begin{aligned}\frac{d\Pi_1}{dk_1} = (&2(0.03k_1^{10} - k_1^{11} - 0.17k_1^9k_2 + 5.5k_1^{10}k_2 + 0.4k_1^8k_2^2 - 12.94.k_1^9k_2^2\\
&- 0.58k_1^7k_2^3 + 17.06k_1^8k_2^3 + 0.52k_1^6k_2^4 - 13.94k_1^7k_2^4 - 0.32k_1^5k_2^5\\
&+ 7.36k_1^6k_2^5 + 0.13k_1k_2^6 - 2.56k_1^5k_2^6 - 0.03k_1^3k_2^7 + 0.56k_1^4k_2^7 + 0.004k_1^2k_2^8\\
&- 0.08k_1^3k_2^8 - 0.0002k_1k_2^9 + 0.007k_1^2k_2^9 - 0.0002k_1k_2^{10} - 2.7\ 10^{-20}k_1^9t\\
&+ 1.4\ 10^{-19}k_1^8k_2t - 0.016k_1^7k_2t + 0.07k_1^6k_2^3t - 0.14k_1^5k_2^4t + 0.14k_1^4k_2^5t\\
&- 0.08k_1^3k_2^6t + 0.025k_1^2k_2^7t - 0.004k_1k_2^8t + 0.0002k_2^9t - 0.125k_1^8k_2^3t^2\\
&- 0.8k_1^4k_2^4t^2 + 0.33k_1^3k_2^5t^2 - 0.08k_1^2k_2^6t^2 + 0.011k_1k_2^7t^2 - 0.0006k_2^8t^2))\\
&/((k_1^2 - 1.25k_1k_2 + 0.25k_2^2)^2(k_1^3 - 1.5k_1^2k_2 + 0.56k_1k_2^2 - 0.06k_2^3)^2) \qquad (22)\end{aligned}$$

$$\begin{aligned}\frac{\partial\Pi_2}{\partial k_2} = (&2(0.008k_1^{14}k_2^4 - 0.06k_1^{13}k_2^5 - k_1^{14}k_2^5 + 0.19k_1^{12}k_2^6 + 6.5k_1^{13}k_2^6\\
&- 0.35k_1^{11}k_2^7 - 18.8k_1^{12}k_2^7 + 0.42k_1^{10}k_2^8 + 32.125k_1^{11}k_2^8 - 0.33k_1^9k_2^9\\
&- 36.2k_1^{10}k_2^9 + 0.18k_1^8k_2^{10} + 28.53k_1^9k_2^{10} - 0.07k_1^7k_2^{11} - 16.27k_1^8k_2^{11}\\
&+ 0.012k_1^6k_2^{12} + 6.85k_1^7k_2^{12} - 0.003k_1^5k_2^{13} - 2.14k_1^6k_2^{13} + 0.0004k_1^4k_2^{14}\\
&+ 0.5k_1^5k_2^{14} - 0.00003k_1^3k_2^{15} - 0.085k_1^4k_2^{15} + 0.00000008k_1^2k_2^{16}\\
&+ 0.01k_1^3k_2^{16}t - 0.0008k_2^2k_2^{17} + 0.00004k_1k_2^{18} - 0.0000001k_2^{19}\\
&+ 0.03k_1^{13}k_2^4t - 0.17k_1^{12}k_2^5t + 0.4k_1^{11}k_2^6t - 0.52k_1^{10}k_2^7t + 0.41k_1^9k_2^8t\\
&-.2k_1^8k_2^9t + 0.053k_1^7k_2^{10}t - 0.004k_1^6k_2^{11}t - 0.0024k_1^5k_2^{12}t + 0.0009k_1^4k_2^{13}t\\
&- 0.0002k_1^3k_2^{14}t + 0.00001k_1^2k_2^{15}t - 0.0000005k_1k_2^{16}t + 0.03125k_1^{12}k_2^4t^2\\
&- 0.11k_1^{11}k_2^5t^2 + 0.14k_1^{10}k_2^6t^2 - 0.05k_1^9k_2^7t^2 - 0.05k_1^8k_2^8t^2 + 0.07k_1^7k_2^9t^2\\
&- 0.04k_1^6k_2^{10}t^2 + 0.01k_1^5k_2^{11}t^2 - 0.003k_1^4k_2^{12}t^2 + 0.0004k_1^3t^2 - 0.00003k_1^2k_2^{14}t^2\\
&+ 0.000001k_1k_2^{15}t^2))/((k_1^3k_2 - 1.5k_1^2k_2^2 + 0.56k_1k_2^3 - 0.06k_2^4)^2\\
&(k_1^4k_2 - 1.75k_1^3k_2^2 + 0.94k_1^2k_2^3 - 0.2k_1k_2^4 + 0.016k_2^5)^2) \qquad (23)\end{aligned}$$

Proposition 1: *(a) If Firm 1 is the foreign firm, a specific tariff on its output leads to an increase in the domestic firm's profits, an increase in the quality of the domestic firm's product, and a decrease in consumer surplus. However national welfare increases since the increase in government revenue offsets the loss in consumer surplus.*

(b) If Firm 1 is the domestic firm, a specific tax (subsidy) on its output leads to a(n) decrease (increase) in its profits, an increase (decrease) in the quality of its product, and a(n) decrease (increase) in consumer surplus. National welfare decreases (increases) since the increase (decrease) in government revenue does not offset the loss (gain) in consumer surplus.

Proof: To obtain various comparative static results with respect to t, we have applied the Implicit Function Theorem to the first order conditions of the profit functions in stage 2.[10] By applying the Implicit Function Theorem to (22) and (23) we get,

$$\begin{bmatrix} \dfrac{\partial g_1(k_1,k_2,t)}{\partial k_1} & \dfrac{\partial g_1(k_1,k_2,t)}{\partial k_2} \\ \dfrac{\partial g_2(k_1,k_2,t)}{\partial k_1} & \dfrac{\partial g_2(k_1,k_2,t)}{\partial k_2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial k_1}{\partial t} \\ \dfrac{\partial k_2}{\partial t} \end{bmatrix} = \begin{bmatrix} -\partial g_1(k_1,k_2,t) \\ -\partial g_1(k_1,k_2,t) \end{bmatrix} \tag{12}$$

The first row of Table 1 summarizes the comparative static results when a tariff/tax is imposed on Firm 1. Note that W_1 describes the overall national welfare when Firm 1 is the foreign firm, and W_2 describes overall national welfare when Firm 2 is the foreign firm. Note the following signs for the derivatives with respect to t.

(i) $\dfrac{d\Pi_2}{dt} > 0$ (ii) $\dfrac{d\Pi_1}{dt} < 0$ (iii) $\dfrac{dCS}{dt} < 0$

(iv) $\dfrac{d\Pi_2}{dt} > 0$ (v) $\dfrac{d\Pi_2}{dt} > 0$

(vi) $W_2 > 0$ (vii) $W_1 < 0$

10. The second order conditions have been verified and they are satisfied.

Table 1
Specific and Ad Valorem Tariffs and Taxes

Policy Instrument	dp_1/dt	dp_2/dt	dq_1/dt	dq_2/dt	$d\pi_1/dt$	$d\pi_2/dt$	dk_1/dt	dk_2/dt	dCS/dt	dG/dt	W_1	W_2
Specific tariff/tax on Firm 1	0.42213	0.57361	-16.634	11.1882	-0.2847	0.01316	0.10843	0.89199	-0.0827	0.2625	0.1929	-0.105
Specific tariff/tax on Firm 2	0.10046	0.54185	10.3914	-97.231	0.13662	-0.1248	-0.2073	0.03006	-0.1421	0.13125	-0.136	0.1258
Ad valorem tariff/tax on Firm 1	-0.0616	0.00607	-0.2351	0.19127	-0.0061	0.00018	-0.0904	0.00958	-0.0044	0.00592	0.0017	-0.005
Ad valorem tariff/tax on Firm 2	0.10046	0.54185	10.3914	-97.231	0.13662	-0.1248	-0.2073	0.03006	-0.1421	0.00048	-0.266	-0.005
$W_1 = d\pi_2/dt + dCS/dt + dG/dt$												
$W_2 = d\pi_1/dt + dCS/dt + dG/dt$												

B. Tariff/Tax on the Firm Producing the Low Quality Product

Once again we consider two situations with respect to optimal policy.

(a) The foreign firm produces the lower quality product and the government imposes a tariff on the foreign firm.

(b) The domestic firm produces the lower quality product and the government imposes a tax on the domestic firm

As in the previous subsection, if $t < 0$, then t can be viewed as a subsidy. Overall welfare is defined as the sum of government revenue, the profits of the domestic firm, and consumers' surplus.

The government revenue function now is given as,

$$G = tq_2 \tag{25}$$

In stage 3, with a tariff/tax on Firm 2, the profit functions become,

$$\Pi_1 = p_1 q_1 - 0.5 k_1 q_1 - k_1^2 \tag{26}$$

$$\Pi_2 = p_2 q_2 - 0.5 k_2 q_2 - t q_2 - k_2^2 \tag{27}$$

Proposition 2: *(a) If Firm 2 is the foreign firm, a specific tariff on its output leads to an increase in the domestic firm's profits, a decrease in the quality of the domestic firm's product, and a decrease in consumer surplus. However national welfare increases, since the increase in government revenue offsets the loss in consumer surplus.*

(b) If Firm 2 is the domestic firm, a specific tax (subsidy) on its output leads to a(n) decrease (increase) in its profits, an increase (decrease) in the quality of the domestic firm's product, and a decrease (increase) in consumer surplus. National welfare decreases (increases) since the increase (decrease) in government revenue does not offset the loss (gain) in consumer surplus.

Proof: Once again to obtain various comparative static results with respect to t, we have applied the Implicit Function Theorem to the first order conditions of the profit functions in stage 2.[11] The second row of Table 1 summarizes the comparative static results when a tariff/tax is imposed on Firm 2.

(i) $\frac{d\Pi_1}{dt} > 0$ (ii) $\frac{d\Pi_2}{dt} < 0$ (iii) $\frac{dCS}{dt} < 0$

(iv) $\frac{dk_1}{dt} < 0$ (v) $\frac{dk_2}{dt} > 0$

(vi) $W_1 > 0$ (vii) $W_2 < 0$

III. Ad Valorem Tariffs and Taxes

In this section we examine the case of ad valorem taxes and/or tariffs being imposed instead of specific taxes/tariffs. We look at the same situations as before, *i.e.* a tax/tariff on Firm 1, and then on Firm 2.

With ad valorem tariffs/taxes, the profit functions in stage 3 for each firm can be written as,

$$\Pi_1 = p_1(1-t)q_1 - 0.5k_1q_1 - k_1^2 \tag{28}$$

$$\Pi_2 = p_2(1-t)q_2 - 0.5k_2q_2 - k_2^2 \tag{29}$$

11. The first order conditions are long and complex. They are available upon request with the authors.

Proposition 3: *(a) When Firm 1 is the foreign firm, an ad valorem tariff on its output leads to an ambiguous effect on the domestic firm's profits and on the quality of its product, and a decrease in the consumer surplus. National welfare increases since the government revenue offsets any decrease in profits or consumer surplus. When Firm 1 is the domestic firm, an ad valorem tax (subsidy) on its output leads to a decrease (increase) in domestic firm's profits, a decrease (increase) in the quality of its product, and a decrease (increase) in consumer surplus. National welfare decreases (increases) since the government revenue (expenditure) does not offset the losses (gains) in profits and consumer surplus.*

(b) When Firm 2 is the foreign firm, an ad valorem tariff on its output leads to an increase in the domestic firm's profits, a decrease in the quality of the domestic firm's product, and a decrease in consumer surplus. National welfare decreases despite the government revenue. When Firm 2 is the domestic firm, an ad valorem tax (subsidy) on its output leads to a decrease (increase) in its profits, an increase (decrease) in the quality of its product, and a decrease (increase) in consumer surplus. National welfare decreases (increases) despite the government revenue (expenditure).

Proof: As in section II, we apply the Implicit Function Theorem to the first order conditions of the profit functions obtained in stage 2. The third and fourth rows of Table 1 indicate the signs of the derivatives with respect to *t*.

Thus with an ad valorem tariff/tax on Firm 1 we have the following:

(i) $\frac{d\Pi_1}{dt} < 0$ (ii) $\frac{d\Pi_2}{dt} > 0$ (iii) $\frac{dCS}{dt} < 0$

(iv) $\frac{dk_1}{dt} < 0$ (v) $\frac{dk_2}{dt} > 0$

(vi) $W_1 > 0$ (vii) $W_2 < 0$

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(vi) $W_1 < 0$ (vii) $W_2 > 0$

IV. Minimum Quality Standards (MQS) on the Foreign Firm

When the domestic government imposes minimum quality standards on the foreign firm, the profit function will reflect fixed values for k. To analyze the effect of minimum quality standards, we assume the same cost functions as in (1) and (2). However the value for the coefficient of (k_q) in the cost function is now assumed to be 0.2.[12] Thus the cost functions are now,

$$c_1(k_1) = 0.2k_1q_1 + k_1^2 \quad (1')$$

$$c_2(k_2) = 0.2k_2q_2 + k_2^2 \quad (2')$$

The profit function when Firm 1 is the foreign firm (i.e k_1 is fixed at k_1^*) will be,

$$\Pi_1 = p_1q_1 - 0.2k_1{}^*k_2 - k_1^{*2} \quad (30)$$

The profit function when Firm 2 is the foreign firm (*i.e* k_2 is fixed at k_2^*) will be,

$$\Pi_2 = p_2q_2 - 0.2k_1k_2^* - k_2^{*2} \quad (31)$$

Proposition 4: *(a) In response to minimum quality standards on Firm 1, the domestic firm (Firm 2) enhances quality, and ends up with higher profits. Consumer surplus also increases, and, as a result, overall national welfare is higher as well.*

(b) In response to minimum quality standards on Firm 2, the domestic firm (Firm 1) enhances quality, but ends up with lower profits. However consumer surplus increases sufficiently so that overall national welfare is higher.

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12. When the cost function coefficient = 0.5 and MQS are imposed on Firm 1 only, the values of the model yield imaginary numbers. To keep the analysis consistent we chose the coefficient value to be 0.2 for both cost functions. Since this is a computational limitation, we expect the values for q, k, Π, CS, and W, to follow the same pattern for all values of the coefficient in the interval [0, 1].

Table 2
Minimum Quality Standards (MQS)

MQS on Firm 1									
k_1 (fixed)	p_1	p_2	q_1	q_2	Π_1	Π_2	CS	k_2	W
0.3	0.17431	0.0073	0.4063	0.2032	–0.044	0.0004	0.0267	0.0187	0.0271
0.4	0.23423	0.0075	0.4048	0.2024	–0.098	0.0004	0.0347	0.019	0.0351
$W = \Pi_2 + CS$									
MQS on Firm 2									
k2 (fixed)	p_1	p_2	q_1	q_2	Π_1	Π_2	CS	k_2	W
0.1	0.03282	0.0239	0.5074	0.2537	–0.009	–0.009	0.0313	0.1181	0.022
0.2	0.02092	0.032	0.5601	0.2801	–0.038	–0.042	0.0667	0.1749	0.0282
$W = \Pi_1 + CS$									

Proof: The proof is from Table 2. The table indicates the values for quantities, quality of the domestic firm, profits, consumer surplus, and overall national welfare, for fixed values of k_1 or k_2. Note that when k_1 is fixed at a higher level, the domestic firm's (Firm 2) profits increase. Consumer surplus and overall welfare are also increasing. When k_2 is fixed at a higher level, the domestic firm's (Firm 1) profits are lower. However consumer surplus and welfare are increasing.

V. Conclusions

The use of industrial organization theory in international trade models has become very relevant. Most of the world trade today is in products produced by oligopolistic industries. Furthermore it is not unusual to see trade in products of varying quality taking place, especially between developed and less developed countries.

We have extended the model of vertically differentiated products to international trade, when a foreign firm producing either a high or a low quality product competes with a domestic firm in the latter's market. We showed the nature of optimal trade policy in this model. Some of the important results we obtain are:

(i) overall welfare increases with a specific tariff on the foreign firm, if it is the one producing the higher quality product, and that this also provides an

incentive for the domestic firm to upgrade the quality of its product. If the domestic firm is the one producing the higher quality product, a subsidy is considered optimal in terms of increasing overall welfare, but leads to a decrease in quality. An ad valorem tariff leads to an ambiguous effect on quality and welfare.

(ii) if a tariff is imposed on the foreign firm when it produces the lower quality product, overall welfare increases, but domestic product quality is reduced. A subsidy for the domestic firm is once again optimal if it produces the lower quality product.

(iii) minimum quality standards imposed on the foreign firm are effective in raising the quality of the domestic product, and increase welfare as well. A policy of minimum quality standards or a specific tariff (when the foreign firm produces the higher quality product), can be considered by governments in developing countries seeking to improve the quality of their domestic products, which typically compete with foreign products of higher quality in the domestic market.

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