

In Search of the Optimum Tariffs Under Third-Degree Price Discrimination

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Abstract

Tariff literature on third-degree price discrimination remains only lightly explored. The purpose of the paper is to investigate whether there exist optimum tariffs in the presence of different demand elasticities. We explore (i) the case of optimum uniform ad valorem tariff, and (ii) the case of optimum discriminatory ad valorem tariffs. A two-stage optimization approach is used: profit is maximized for a foreign firm in the first stage and importing country's optimum tariff rates are determined under maximization of welfare (sum of consumer surplus and tariff revenue). The result indicates that the optimum tariffs may not exist even with a set

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of well-behaved linear demand functions. Consequently, computer simulations are needed with cost and demand parameters to ensure better welfare position for home country.

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- **Key words:** Optimum, Discriminatory Tariffs

I. Introduction

Enough ink has been spread on third-degree price discrimination since the work by Pigou (1920). Perhaps the most well-known piece is the one by Robinson (1934) in which output and welfare effects were explored. Subsequent work has been advanced by Edwards (1950), Silberberg (1970), Battalio and Ekelund (1972), Yamey (1974), Finn (1974), Greenhut and Ohta (1976), Smith and Formby (1981), Schmalensee (1981), Formby et al. (1982), Varian (1985), Shin et al. (1988), Nahata *et al.* (1990), Schwartz (1990), Malueg (1993). The application of third-degree price discrimination is widespread even across different continents over various types of commodities (e.g., academic journal of Europe and North America according to Hamaker and Astle (1984)). Nonetheless, the taxation or tariff topic has been much ignored (e.g., Yang, 1993; Cheung, 1998). Yang's work primarily focuses on single taxes within the framework of profit maximization. Recently, Yang et al. (1996) prove that an optimum uniform tariff over n countries must be a convex combination of different discriminatory tariffs with a set of linear demand functions. However, their tariffs are measured in terms of dollar per unit or unit tariffs. Not surprisingly, tariffs of different countries may vary substantially according to domestic production and consumption environments. For instance, average tariff rates on toys were 3.2 per cent, 4.6 per cent, 8 per cent, 4.3 per cent, and 41.6 per cent for Canada, Australia, South Korea, Japan, and China respectively in 2000, 18.2 per cent, 3 per cent, 30.2 per cent, 16 per cent, and 40 per cent for food items, 2.97 per cent, 1.4 per cent, 5 per cent, 1.4 per cent, 24.7 per cent, for jewelry and 4.8 per cent, 7.3 per cent, 7.1 per cent, 2.1 per cent, and 25.8 per cent for automobile. It is to be pointed out that discriminatory tariffs under other imperfect competition (Brander and Spencer, 1984; Hwang and Mai, 1991) begin to receive more attention. For instance, Li et al. (1996) simulated pollution tariffs using the Hwang-Mai model who based their analysis on the Cournot competition. Chou et al (2002) apply the Hwang-Mai model to a multi-country

pollution tariff coordination scheme: lower tariff for countries with more rigorous anti-pollution standards. Clearly the Cournot models have yielded more empirical applications, for their solution sets incorporate that of traditional mathematical programming model. For example, Yang et al. (2002) proposed and implemented a spatial equilibrium Cournot model with taxes in the framework of linear complementarity programming which admits more sets of solutions than are allowed by nonlinear concave programming. It is long-recognized that nonlinear mathematical programming is a special case of the linear complementarity programming (Cottle, 1972; Irwin and Yang, 1982, 1983). As is to be shown in this paper, the third-degree price discrimination model is cast within a two-stage optimization framework in which the solution of the first stage is fed into the second stage as input before obtaining the optimum tariffs. The filtering procedure may be quite restrictive regarding the optimum tariffs. As a consequence, we employ highly simplified linear demand and cost functions to demonstrate that numerical simulations may well be necessary in the case when optimality is either lacking or not feasible, or being forced to take on corner solutions (zero consumptions for some countries). Given the prevalence of different tariff rates, the purpose of this paper is to investigate the possibility of an optimum ad valorem tariff, which involves changes in both intercept and slope of a demand function. To the best of our knowledge, the topic remains largely unexplored and the result is used to fill a void in the literature. The organization of the paper is as follows. The next section presents optimum uniform ad valorem tariff. Section III discusses the possibility of discriminatory ad valorem tariff. Section IV includes a conclusion.

II. Optimum Uniform Ad Valorem Tariff

Consider a demand function for market i (for $i=1, \dots, n$) $q_i = g(p_i)$ in which q_i is a monotonically decreasing function of p_i so that $p_i = f(q_i)$ is uniquely defined. The firm's total cost function is TC such that $TC'(q) \geq 0$, i.e., marginal cost is nonnegative. Note that $q = \sum q_i$, we use the two-stage optimization procedure in which the firm's after-tariff profit is maximized or

$$\text{Maximize } \pi = (1 - \mu) \sum p_i q_i - TC(q) \quad (1)$$

where μ is the single ad valorem tariff, the first-order condition leads to

$$\frac{\partial \pi}{\partial q_i} = (1 - \mu)(p_i + q_i p'_i) - \frac{\partial TC}{\partial q} \frac{\partial q}{\partial p_i} = 0 \quad \forall i \in N$$

or $(1 - \mu)MR_i - MC = 0$ (2)

where N is an integer set, and $\partial q / \partial q_i = 1$.

We then solve for n q_i^* s from n first-order conditions. If the solution of q_i^* s exists uniquely, it is a function of μ and demand function parameters or

$$q_i^* = h(\mu, \alpha) \quad (3)$$

where α is a set of estimated coefficients of demand functions.

The second stage involves the choice of an optimal μ that maximizes the sum of consumer surplus (CS) and tariff revenue (TR):

$$\text{Maximize } V = CS + TR \quad (4)$$

$$\text{where } CS = \sum (\int_0^{q_i^*} p_i(q_i) dq_i - p_i q_i^*) \quad (5)$$

$$TR = \mu \sum p_i q_i \quad (6)$$

q_i^* = optimal output in market i under the single ad valorem tariff.

The first order condition is

$$\sum \partial \int_0^{q_i^*} p_i(q_i) dq_i / \partial q_i \cdot \frac{\partial q_i}{\partial \mu} - \sum p_i \frac{\partial q_i}{\partial \mu} - \sum q_i^* \frac{\partial p_i}{\partial q_i} \frac{\partial q_i}{\partial \mu} = 0 \quad (7)$$

Since q_i^* s are determined from the first stage, $q_i^* = h(\mu, \alpha)$ can also be evaluated. Substitute them into (7) and one may solve for an optimal μ in the case of single ad valorem tariff. However, equation (7) assumes a general functional form whose components are determined from the first-stage optimization. In other words, the values assumed from the first stage can be rather restrictive, and as such one may have only sub-optimal values for the uniform ad valorem tariff. To simplify the calculation, we employ separable linear demand functions for only two markets and a linear total cost function for the firm as shown below:

$$p_i = a_1 - b_1 q_1 = 225 - 0.005 q_1$$

$$p_2 = a_2 - b_2 q_2 = 125 - 0.00125 q_2$$

$$TC = c + d(q_1 + q_2) = 2000 + 25 (q_1 + q_2)$$

The profit-maximizing quantities from the first stage optimization can be shown as

$$q_1^* = \frac{a_1(1-\mu) - d}{2b_1(1-\mu)} \quad (8)$$

$$q_2^* = \frac{a_2(1-\mu) - d}{2b_2(1-\mu)} \quad (9)$$

substituting (8) and (9) and $\partial q_1^*/\partial \mu$ and $\partial q_2^*/\partial \mu$ into $\partial V/\partial \mu = 0$ (or equation (7)),

we obtain

$$\begin{aligned} & -(a_1^2 b_2 + a_2^2 b_1) \mu^3 + (3a_1^2 b_1 + 3a_2^2 b_1) \mu^2 - (3a_1^2 b_2 + 3a_2^2 b_1 + a_1 b_2 d \\ & + a_2 b_1 d + 3b_1 d^2 + 3b_2 d^2) \mu + (a_1^2 b_2 + a_2^2 b_1 - a_1 b_2 d - a_2 b_1 d) = 0 \end{aligned} \quad (10)$$

Evidently, the search for the optimal uniform ad valorem tariff hinges on the solution of this rather complicated polynomial equation of degree three, whose solution (exactly three) characteristics are contained in the well-known De Cartes's theorem regarding the exact number of solutions and the sign test for positive (negative)-valued solutions.

A simulation by a nonlinear optimization GINO (Liebman et al. 1986) indicates the optimum uniform ad valorem tariff of 1.212: obviously an infeasible one (it exceeds 100 per cent). The other two solutions are a pair of complex conjugates, irrelevant for our analysis. *Note that lacking the optimum solution does not imply a sub-optimal solution does not exist.* For this reason, we simulated the tariff rate from 0 per cent to 90 per cent with an increment of 10 percentage points and evaluate the welfare in terms of $CS+TR$. Results are reported in Table 1.

An examination of Table 1 indicates the sum of CS and TR is maximized approximately at $\mu=60$ per cent, indicating a rather stiff tariff rate. Of course, parameters of the demand and cost functions are arbitrarily selected, one should not assign too much empirical relevance to the simulated solution. In addition, one must bear in mind that the producer and consumers may not reside in the same country, hence

Table 1. Welfare under Uniform Ad Valorem Tariff

μ	0	0.1	0.2	0.3	0.4
$CS+TR$	2,000,000	2,463,965	2,899,804	3,295,169	3,623,611
μ	0.5	0.6	0.7	0.8	0.9
$CS+TR$	3,843,749	3,858,593	3,419,444	1,649,999	0

the summation of π and CS and TR , unlike welfare measurement within an economy, does not have much relevance. The crux of the matter is that the optimum uniform ad valorem tariff may well not exist even in the case of simple demand and cost functions for two markets. A simulation is needed to ferret out sub-optimal values.

III. The Optimal Discriminatory Tariff

Consider a set of discriminatory tariff μ_i imposed in the i the market of a region. The after-tariff profit function is therefore

$$\pi = n \sum_{i=1}^n (1 - \mu_i) p_i q_i - TC(q) \tag{11}$$

Profit-maximization of (11) with respect to $q = (q_1 \dots q_n)$ leads to

$$\frac{\partial \pi}{\partial q_i} = (1 - \mu_i) \cdot (p_i + q_i + p_i') - TC'(q) = 0 \tag{12}$$

for all $i \in N$. There are n such equations for $q = (q_1^* \dots q_n^*)$. Assuming a unique solution set exists over R_+^n , we can express $q_i^* = k(\beta, \mu)$ where β is a vector of estimated demand function parameters and $\mu = (\mu_1 \dots \mu_n)$. The objective function of the second-stage optimization is the sum of consumer surplus and tariff revenue ($V = CS+TR$) or

$$\text{Maximize } V = CS + TR = \sum (\int_0^{q_i^*} p_i dq_i - p_i q_i^*) + \sum \mu_i p_i q_i^* \tag{13}$$

Again the first-order condition yields immediately.

$$\frac{\partial V}{\partial \mu_i} = \frac{\partial (\int_0^{q_i^*} p_i)}{\partial q_i} \cdot \left(\frac{\partial q_i}{\partial \mu_i} \right) - \left(\frac{\partial p_i q_i^*}{\partial \mu_i} \right) + p_i q_i^* + \mu_i p_i \left(\frac{\partial q_i^*}{\partial \mu_i} \right) + \mu_i q_i^* p_i' \left(\frac{\partial q_i^*}{\partial \mu_i} \right) = 0 \tag{14}$$

for all $i \in N$

There are n such equilibrium conditions. Substituting $q_i^* = k(\mu, \beta)$ and $\partial q_i^* / \partial \mu_i^*$ into (14), we can readily obtain a system of n nonlinear equations. While the existence of the solution rests in the domain of the fixed point theorem, it generally is difficult to calculate a set of global optimal solution especially when n is large. With the identical linear separable demand and cost functions, we obtain the optimum quantity for each market:

$$q_1^* = \frac{a_1(1 - \mu_1) - d}{2b_1(1 - \mu_1)} \quad (15)$$

$$q_2^* = \frac{a_2(1 - \mu_2) - d}{2b_2(1 - \mu_2)} \quad (16)$$

substituting (15), (16) and $\partial q_i^* / \partial \mu_i^*$ into (14) and rearranging leads to

$$\frac{\partial V}{\partial \mu_1} = \frac{1}{4b_2(1 - \mu_1)^3} [-a_1d(1 - \mu_1) + d^2 + a_1^2(1 - \mu_1)^3 - d^2(1 - \mu_1) - 2\mu_1d^2] = 0 \quad (17)$$

which produces the polynomial equation of degree 3 or

$$a_1^2\mu_1^3 - 3a_1^2\mu_1^2 + (3a_1^2 + a_1d + d^2)\mu_1 + (-a_1^2 + a_1d) = 0 \quad (18)$$

in which (a_1, a_2) and (b_1, b_2) , denote intercepts and slopes of the two linear demand functions; c and d are intercept and slope of the cost function; μ_1, μ_2 represent discriminatory tariff rates in two consumption countries.

Following exactly the same procedure, we obtain the following polynomial equation for market 2:

$$a_2^2\mu_2^3 - 3a_2^2\mu_2^2 + (3a_2^2 + a_2d + d^2)\mu_2 + (-a_2^2 + a_2d) = 0 \quad (19)$$

In this simplified example, we find the optimal discriminatory tariffs are $\mu_1 = 1.341$ and $\mu_2 = 1.266$. The other solutions are complex conjugates and hence ignored. Since lacking a global solution does not translate into giving up the

Table 2. The Welfare under Discriminatory Tariff

μ_1	μ_2	0.1	0.3	0.5	0.7	0.8
0.1		2,463,965	2,878,252	3,211,033	3,272,838	2,892,282
	0.3	2,879,884	3,294,132	3,626,912	3,688,718	3,308,162
	0.5	3,096,676	3,510,967	3,843,748	3,905,553	3,524,999
	0.7	2,610,571	3,024,858	3,357,638	3,419,447	3,038,888
	0.8	1,221,682	*1,635,969	*1,968,750	*2,030,555	*1,649,999

denotes q_1^ , therefore a corner solution

search, we employ a simulation of 25 computer runs with $\mu_i = 0.1, 0.3, 0.5, 0.7$ and 0.8 via GINO to obtain the welfare measure ($CS+TR$). Note that we do not raise the tariff to 0.9 where $q_i^* = 0$ for both markets. The results are reported in Table 2.

An inspection of Table 2 reveals readily that locally optimal pair of discriminatory tariffs lies around $(\mu_1=0.5, \mu_2=0.7)$. Note that numerical values along the diagonal in Table 2 are the welfare measure under uniform tariffs. Clearly it needs no mathematical proof to show that uniform tariff taxation cannot be superior to the discriminatory one since it is only a subset of the latter. Note that our simulation is based on two arbitrary chosen linear demand and a simplified linear cost function. As such, tariff rates between 50 per cent and 70 per cent are not empirically relevant. We must be on our guard, against the possibility of such prohibitive rates for they are politically infeasible, should such rates appear as optimum solution. However, empirical estimations of the model remain an interesting topic for future research.

IV. The Optimal Discriminatory Tariff with a Fixed Tariff Revenue

More often than not, a government policy is set to meet a predetermined level of tariff revenue while maximizing the consumers' welfare (consumer surplus). The first-stage optimization is the same as that in the previous section or

$$\text{maximize } \pi = \sum_{i=1}^n (1 - \mu_i)p_i q_i - TC(q) \tag{20}$$

with the resulting optimal quantity for $n = 2$ (linear case):

$$q_1^* = \frac{a_1(1 - \mu_1) - d}{2b_1(1 - \mu_1)} \tag{21}$$

$$q_2^* = \frac{a_2(1 - \mu_2) - d}{2b_2(1 - \mu_2)} \quad (22)$$

The second-stage optimization procedure can be cast in the following:

$$\text{Maximize } CS = \sum_0^{q_i^*} P_i dq_i - P_i^* q_i^* \quad (23)$$

$$\text{Subject to } \sum \mu_i P_i q_i = R_o \quad (24)$$

The Lagrange function L and its first-order condition for the linear case ($n=2$) can be shown as $L = CS + \lambda(R_o - \sum \mu_i P_i q_i)$

$$\frac{\partial L}{\partial \mu_1} = b_1 q_1 \frac{\partial q_1}{\partial \mu_1} + \lambda \left[-(a_1 q_1 - b_1 q_1^2) - \mu_1 (a_1 - 2b_1 q_1) \frac{\partial q_1}{\partial \mu_1} \right] = 0 \quad (25)$$

$$\frac{\partial L}{\partial \mu_2} = b_2 q_2 \frac{\partial q_2}{\partial \mu_2} + \lambda \left[-(a_2 q_2 - b_2 q_2^2) - \mu_2 (a_2 - 2b_2 q_2) \frac{\partial q_2}{\partial \mu_2} \right] = 0 \quad (26)$$

$$\frac{\partial L}{\partial \lambda} = R_o - \mu_1 (a_1 q_1 - b_1 q_1^2) - \mu_2 (a_2 q_2 - b_2 q_2^2) = 0 \quad (27)$$

Substituting $\frac{\partial q_1^*}{\partial \mu_1}$ and $\frac{\partial q_2^*}{\partial \mu_2}$ and q_1^*, q_2^* into (25) and (26), we have

$$-a_1(1 - \mu_1)d + d^2 + \lambda[-a_1^2(1 - \mu_1)^3 + d^2(1 - \mu_1) + 2\mu_1 d^2] = 0 \quad (28)$$

since $4b_1(1 - \mu_1)^3 \neq 0$ and

$$-a_2(1 - \mu_2)d + d^2 + \lambda[-a_2^2(1 - \mu_2)^3 + d^2(1 - \mu_2) + 2\mu_2 d^2] = 0 \quad (29)$$

since $4b_2(1 - \mu_2)^3 \neq 0$.

From (28) it can be shown

$$\lambda = \frac{-a_1(1 - \mu_1)d + d^2}{a_1^2(1 - \mu_1)^3 - d^2(1 - \mu_1) - 2\mu_1 d^2} \quad (30)$$

where λ is the Lagrange multiplier.

Likewise from (29), we have

$$\lambda = \frac{-a_2(1-\mu_2)d + d^2}{a_2^2(1-\mu_2)^3 - d^2(1-\mu_2) - 2\mu_2 d^2} \quad (31)$$

Equating (30) to (31) yields readily

$$\frac{-a_1(1-\mu_1)d + d^2}{a_1^2(1-\mu_1)^3 - d^2(1-\mu_1) - 2\mu_1 d^2} = \frac{-a_2(1-\mu_2)d + d^2}{a_2^2(1-\mu_2)^3 - d^2(1-\mu_2) - 2\mu_2 d^2} \quad (32)$$

or

$$\begin{aligned} & -a_1 a_2^2 (1-\mu_1)(1-\mu_2)^3 - a_2 a_1^2 (1-\mu_2)(1-\mu_1)^3 \\ & + d^2 (a_1 - a_2)(1-\mu_1)(1-\mu_2) + 2d^2 [a_1 \mu_2 (1-\mu_1) \\ & - a_2 \mu_1 (1-\mu_2)] + d [(-a_1^2 (1-\mu_1)^3) + a_2^2 (1-\mu_2)^3] \\ & + d^3 (\mu_1 - \mu_2) = 0 \end{aligned} \quad (33)$$

The polynomial equation coupled with equation (29) can be quite involved (negative solutions) and as such we simulate the discriminatory optimal tariff scheme by varying μ_1 and μ_2 with an increment of 0.1 and setting $R_0 = 500,000$. The result is reported in Table 3. It is to be noted that the lower part of the table (with *) indicates a corner solution in which $q_2 = 0$ while the upper part (with **) represents cases of $q_1 = 0$. These corner solutions are not considered in the above optimum conditions in which we assume positive quantities. An examination points to the direction of $\mu_1 = \mu_2 = 0.1$: the lowest tariff rate which maximizes consumers surplus while meeting the target of the tariff revenue of 500,000 in our simulation. It speaks to the fact that lower tariff rates are consistent with the consumers' welfare objective: lower prices and greater amount of consumer surplus. The result of the simulation carries an important message: direct optimization in third-degree price discrimination model may well not yield a feasible solution. For a given set of demand and cost estimates, computer simulations, which produce local optimal solutions, seem to serve better purpose than does the direct two-stage optimization method. In the wake of rising regional economic integration in which reciprocal tariffs are more likely to prevail, a low uniform tariff is preferred. Outside the region, however, a set of discriminatory tariffs can be calculated for interregional policy evaluations. In a nutshell, as this paper has indicated, traditional optimization techniques may not be appropriate

Table 3. Consumer Surplus Under Discriminatory Tariff With a Fixed Tariff Revenue

$\mu_1 \backslash \mu_2$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1,798,991	1,246,442	1,319,678	1,379,762	1,405,288	1,422,018	1,433,675	1,442,298	1,448,936
0.2	1,441,410	833,158	907,665	977,457	977,457	977,457	977,457	977,457	977,457
0.3	1,449,062	1,236,498	718,566	656,855	656,855	656,855	656,855	656,855	656,855
0.4	1,458,377	1,340,593	718,566	605,371	579,385	579,385	579,385	579,385	579,385
0.5	1,465,216	1,385,916	718,566	605,371	562,499	548,058	548,058	548,058	548,058
0.6	1,470,229	1,411,199	718,566	605,371	562,499	541,457	532,243	532,243	532,243
0.7	1,474,017	1,427,320	718,566	605,371	562,499	541,457	529,534	517,848	523,138
0.8	1,476,966	1,438,490	718,566	605,371	562,499	541,457	529,534	522,127	508,860
0.9	1,479,322	1,446,685	718,566	605,371	562,499	541,457	529,534	522,127	517,185

when the optimum tariffs are considered either prohibitively high or being forced to take on corner solutions. Or worse, they simply do not exist.

V. Conclusion

In this paper, we have shown that the optimal ad valorem tariffs under third-degree price discrimination could be quite illusive. In the case of uniform tariff with linear separable demand and linear cost functions for just two markets, the search for the optimality is contained in De Carte's theorems regarding existence and characteristics of solutions. In the cases of discriminatory tariffs, the existence of optimal tariff is housed in the domain of fixed point theorem regarding a system of general nonlinear equations. With the same parameters used in our simulations, it is found that globally optimal tariffs do not exist in these cases. It is to be pointed out that linear functions are used to illustrate the potential lack of the solution, rather than its existence. To this end, computer simulations are employed to search for sub-optimal values even in the case of linear separable demand and cost functions for $n=2$. Clearly, an optimum uniform tariff is only a special case of optimum discriminatory tariffs. One can also eliminate prohibitively high tariffs for being infeasible (corner solution). Within the economically integrated region (e.g., NAFTA, EU, ASEAN), one can concentrate on the size of consumer surplus in

addition to a fixed tariff revenue on the diagonal line (uniform tariff). Our simulation reveals a low tariff rate (but not zero) is preferred. For trades outside the region, the entire table barring the corner solutions needs to be evaluated in terms of consumer surplus along with a target tariff revenue for each country.

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